STATISTICALLY-BASED REORIENTATION OF DIFFUSION TENSOR FIELD

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ABSTRACT

This paper presents a new method for warping of diffusion tensor fields. The proper reorientation of a tensor field requires knowledge of the underlying fiber orientation, which is not known *a priori*. Accordingly, a probabilistic representation of the fiber direction is adopted and used in a Procrustean estimation of the rotational component of the warping field. The estimated rotational component, along with the warping field, are then used to warp and properly reorient a tensor field. Results from diffusion tensor MR images demonstrate a dramatic improvement of signal to noise ratio after statistical averaging of properly registered tensor fields across 9 individuals.

1. INTRODUCTION

Diffusion tensor imaging (DTI) is a relatively new magnetic resonance imaging method, which is based on measurements of microscopic diffusion of water molecules that presumably indicate the direction of nerve bundles. Its simplest and most commonly application is based on measuring water diffusion along 6 independent directions, thereby forming a tensor for each voxel in the image. The principal eigenvector of this tensor that corresponds to the largest eigenvalue indicates the direction along which water diffuses more freely. Brain tissue with oriented structure, such as fiber bundles, displays high degree of anisotropy in water diffusion. Several measures of anisotropy have been proposed in the literature [1, 2], based on the eigenvalues of the diffusion tensor. The most commonly used measures are *fractional anisotropy (FA)*, *relative anisotropy (RA)*, and *volume ratio (VR)*[1].

Based on DTI, one can parcelate the white matter into different regions, according to measurements of the magnitude and direction of anisotropy. DTI can also be used for understanding connectivity in the brain [3, 4, 5, 6, 7]. However, DTI's signal to noise ratio is relatively low. Our premise in this paper is that ratio of signal to noise, and consequently fiber tracking, can be significantly improved if data across individuals are pooled together and averaged, or somehow considered jointly in a statistical model seeking group differences or correlations with clinical variables. A key step to make the data comparable is spatial normalization. In this paper, we present a new algorithm based on Procrustean estimation to achieve this key step and therefore resolve these problems. This is not a trivial extension of deformable registration of scalar fields, such as MR images, for reasons explained in the following section. Similar work was presented recently in [8], where two methods, ”FS method” and ”PPD”, were presented. However, both methods assumes that the spatial transformation is relatively small (see deduction in [9, 8]). Moreover, the second method is sensitive to errors in a noisy environment.

In Section 2, we first present some preliminaries on diffusion tensors and warping strategies, along with a more detailed reference to related work. In Section 3 we present our approach to spatial transformation of tensor fields. In Section 4, we present an experiment with real images and demonstrate that signal to noise ratio can be significantly improved by properly averaging tensor fields across individuals, which untimately leads to more reliable fiber tracking. Finally, we conclude in Section 5.

2. PRELIMINARIES

Spatial transformations of tensor fields pose difficulties not previously considered in deformable registration methods of scalar images. Since the measurement of tensors reflects the tissue's micro structure, spatial transformation does not change the physical properties measured by DTI. Therefore, directly applying the spatial transformation to warp the tensor ellipsoid would be wrong. A simple example is a spatial transformation of scaling when a tissue is stretched. The fiber becomes thicker, which only means to increase the number of tensors proportionally to the thickness, but the tensor's shape should not change, since it reflects the anisotropy of the fiber. A tensor bears an orientation indicating the underlying tissue anisotropy. Therefore, compared with deforming a scalar image, DTI warping requires the reorientation of the tensor, in addition to relocating it into the normalized space.

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No matter what the reorientation strategy of a tensor field is, for a given spatial transformation, it is important to note that this reorientation strategy should not depend only on the spatial transformation itself, but also on the orientation of the underlying fibers. This can be puzzling, at first, but it is schematically explained in Fig. 1 (see also [10, 8]). Rigid translation, rotation, and scaling will not have any effect on a tensor's relative orientation. But shear consists of stretch and rotation components, and the rotation will differ for differently oriented tensors. Suppose there is a horizontally applied force as illustrated in Fig. 1. The shearing effect introduces a rotational component to any tensor that is not exactly horizontally oriented. For example, this rotational component effect will tilt vertically aligned tensors (perpendicular to the force) in Fig. 1, while the orientation of the horizontally aligned tensors remains intact.

Accordingly, the reorientation of a tensor field should ideally be based on knowledge of the orientation of the underlying fiber pathways. If this is the case, we can easily adjust the tensors in the warped image to align them with their corresponding fiber pathways. But, these pathway orientations are not known and are exactly what we are trying to estimate.

To overcome this limitation, we have developed an approach that assumes that the true underlying fiber orientation follows a statistical distribution that can be estimated from tensor measurements in the neighborhood of the voxel under consideration. Accordingly, the reorientation of the tensor is determined by the estimator that finds the optimal rotation, in a statistical fashion.

3. PROCRUSTEAN TENSOR REORIENTATION

3.1. Estimating the rotation matrix

Proper reorientation of a tensor field requires the knowledge of the fiber's orientation (see Fig. 1). When there is no noise in the tensor measurements, it would be straightforward to find the rotation matrix **U**. It is the matrix that maps the primary direction (PD) of the tensor at a particular location to its deformed configuration by the displacement field. This was effectively the assumption made in [8]; however, it is sensitive to errors in estimating the primary and/or secondary eigenvector(s). Let $\mathbf{x}$ be the coordinates of a voxel under consideration, and $\mathbf{D}$ the corresponding tensor measured at that location. Assume that the orientation of the underlying fiber passing through $\mathbf{x}$ is known and given by vector $\mathbf{v}$. Let the warping transformation map the vector $\mathbf{v}$ to a vector $\mathbf{v'}$. This transformation (rotation) can be represented by a unitary matrix $\mathbf{U}(\mathbf{v})$, which can then be applied to $\mathbf{D}$, reorienting it to $\mathbf{U}(\mathbf{v})\cdot\mathbf{D}\cdot\mathbf{U}^T(\mathbf{v})$. Note that the shape of the tensor does not change in this case; its eigenvectors are simply rotated by $\mathbf{U}(\mathbf{v})$. However, in reality we do not know the true underlying fiber orientation. The PD of the tensor is only a noisy observation of the fiber direction. Directly using this noisy observation [8] could adversely affect the accuracy of reorientation. Let's assume, temporarily, that we know the probability density function (*pdf*) of the fiber's orientation around a particular voxel. Then, we can perform random sampling of this distribution, each time generating a vector **v**. For each such vector, its deformed configuration can be readily determined via the displacement field. Therefore, this procedure would result in a number of orientations and their respective deformed configurations. One can then determine the rotation matrix that best fits this set of measurements.

Suppose that the *pdf* of the vector **v** is given by $f(\mathbf{v})$. Then we are seeking matrix **U** that minimizes:

$$\min_{\mathbf{U}}\left(\int_{\mathbf{v}} f(\mathbf{v})\|\mathbf{v'}-\mathbf{U}\cdot\mathbf{v}\|^2 d\mathbf{v}\right) \tag{1}$$

The solution to the discrete version of this problem is well known and is called the Procrustean estimation [11]. If vectors **v** are concatenated to a matrix $\mathbf{B}$ and vectors **v** to another matrix $\mathbf{B}$, **U** can then be obtained by the singular value decomposition (SVD) of $\mathbf{A}\cdot\mathbf{B^T}$: $\mathbf{U}=\mathbf{V}\cdot\mathbf{W^T}$, $\mathbf{A}\cdot\mathbf{B^T}=\mathbf{V}\cdot\mathbf{\Sigma}\cdot\mathbf{W^T}$. This minimizes:
$\sqrt{\|\mathbf{A}\|^2+\|\mathbf{B}\|^2+2\sum_{\mathbf{i=1}}^{\mathbf{m}}\sigma_{\mathbf{i}}(\mathbf{AB^T})}$
where $\sigma_i(S)$ is the singular value of matrix S.

The effect of Procrustean estimation can actually be interpreted as an approximate transformation (rotation, scaling) that minimizes the difference of point coordinates-pairs in $\mathbf{A}$ and $\mathbf{B}$. When we use *vectors that are normalized* to form $\mathbf{A}$ and $\mathbf{B}$, Procrustean estimation actually results in a pure rotation for the vectors.

In summary, if a statistical representation of the underlying fiber's orientation is not known via $f(\cdot)$, then the matrix **U** can be determined in statistical fashion. It is the matrix that best approximates the rotational component of the displacement field along the direction implied by $f(\cdot)$.

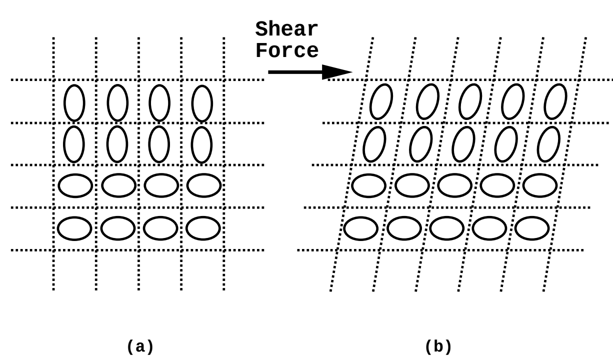

Fig. 1. *Tensor reorientation depends on both spatial transformation and its original orientation. (a) Original tissue; (b) Tensor's proper reorientation during horizontal shearing.*

3.2. Neighborhood sampling strategy

Unfortunately, in practice we do not have knowledge of the *pdf* of the tensor measurements around each voxel. To solve this problem, we consider a small neighborhood $\mathcal{N}(\mathbf{x})$ around location $\mathbf{x}$, from which we draw samples from $f(\cdot)$. In particular, it is reasonable to assume that if neighborhood $\mathcal{N}(\mathbf{x})$ is relatively small around $\mathbf{x}$ that, the direction of the underlying fiber remains approximately constant, and any variations of the samples in the direction of the primary eigenvector measured at points within $\mathcal{N}(\mathbf{x})$ reflect the variations implied by $f(\cdot)$. In order to generate random samples of $\mathbf{v}$ drawn from $f(\cdot)$, we could then attempt to estimate $f(\cdot)$, perhaps by assuming a Gaussian form or some other distribution form, from samples obtained from $\mathcal{N}(x)$, and then apply random sampling techniques to generate a large number of samples. This method would depend on an arbitrary assumption of the distribution, and would be time consuming in random resampling. Alternatively, we can directly use the samples in $\mathcal{N}(\mathbf{x})$, thus eliminating the weakness of assuming certain type of distribution. A drawback is that the number of samples it can generate may be limited, as it depends on the size of the small neighborhood. Our estimator for the rotation matrix at location $\mathbf{x}$ would then be determined via the following procedure:

Step A.1 *Estimate local neighborhood* $\mathcal{N}(\mathbf{x})$.

Step A.2 *Based on the existing observations, use the unit vectors in* $\mathcal{N}(\mathbf{x})$ *as random samples* $\mathbf{v}$*, forming the matrix* $\mathbf{B}$.

Step A.3 *For each sample,* $\mathbf{v}$*, find its configuration after applying the displacement field, and normalize it to a unit length vector,* $\mathbf{v}'$*. Use these vectors to form the columns of the other matrix* $\mathbf{A}$ *after the spatial transformation.*

Step A.4 *Compute transformation matrix* $\mathbf{U}$ *using Procrustean estimation, so that,* $\mathbf{U}$ *minimizes* $\|\mathbf{A} - \mathbf{U} \cdot \mathbf{B}\|^2$.

With the samples from $\mathcal{N}(\mathbf{x})$ effectively replacing the *pdf* $f(\mathbf{v})$, equation (1) becomes:

$$\min_{\mathbf{u}}\left(\int_{\mathcal{N}(\mathbf{x})} \|(\mathbf{v}'(\mathbf{y}) - \mathbf{U} \cdot \mathbf{v}(\mathbf{y})\|^2 d\mathbf{y}\right) \qquad (2)$$

We have assumed the samples in the neighborhood follow the $f(\cdot)$, therefore, $\mathcal{N}(\mathbf{x})$ should not include heterogeneous regions or differently oriented fibers, since that would violate our assumption that the $f(\cdot)$ is approximately constant with $\mathcal{N}(\mathbf{x})$. Since white matter fibers are physically very thin and elongated structures, in order to achieve a relatively large neighborhood, while restricting it to the fiber, we use an ellipsoidal neighborhood, whose shape is dictated by the shape of the tensor field around $\mathbf{x}$. The neighborhood is determined according to the following iterative procedure:

Step B.1: *Select the value,* $\mathcal{V}$*, of the neighborhood from which samples are drawn.*

Step B.2: *Determine the ellipsoidal neighborhood,* $\mathcal{N}(x)$*, around* $\mathbf{x}$*, that has volume* $\mathcal{V}$*, and axis ratio equal to that of the eigenvalues of the tensor at* $\mathbf{x}$.

Step B.3: *Find the average,* $\bar{\mathbf{D}}$*, of the tensor field within* $\mathcal{N}(x)$.

Step B.4: *Update the ellipsoidal neighborhood,* $\mathcal{N}(x)$*, around* $\mathbf{x}$*, that has volume* $\mathcal{V}$*, and axis ratio equal to that of the eigenvalues of* $\bar{\mathbf{D}}$.

Step B.5: *If* $\mathcal{N}(x)$ *is unchanged, stop; else, go to Step B.3.*

The volume $\mathcal{V}$ of the ellipsoidal neighborhood and the resampling density are kept constant during this procedure, to warranty that the same number of samples of the vector $\mathbf{v}$ is used for each voxel in the image.

4. EXPERIMENT

In this experiment, we used 9 normal subjects to demonstrate that statistical averaging on tensor fields after deformable registration and appropriate reorientation improves DT image quality. Among the 9 subjects, we randomly selected one as the template and warped the rest 8 to this selected one. Co-registered with each DT image was a standard T_1-weighted image obtained during the same session by using UCLA's AIR program [12, 13].

In order to obtain the displacement field, we first applied the spatial normalization method described in [14] and referred to as Hierarchical Matching Mechanism for Elastic Registration (HAMMER) to the T_1 images, and then applied the estimated deformation field to the DT images. The appropriate rotation field was calculated from the displacement field, using the method of this paper. Fig. 2 is a frontal view of a whole DT brain image, displaying the PD colormap of one of the 9 individuals, which is extremely noisy. Fig. 3 is the same frontal view of the average result of 9 subjects, where most of noise has been reduced allowing for the main fiber bundles to appear. Fig. 4 is a typical coronal slice of this average result. The color coding of tensor directionality is explained in the figure's caption.

5. CONCLUSIONS

In this paper, we have developed a new method based on Procrustean estimation for warping an arbitrarily deformed DT images. It does not require the displacement to be relatively small, nor does it assume that the fiber direction is known and/or that it coincides with the first principal eigenvector of the diffusion tensor. Our method is based on a statistical estimation in a properly shaped neighborhood, therefore it is relatively robuster.

We note that data averaging can increase the signal to noise ratio only if 1) tensor fields are well registered after spatial normalization and 2) the tensor fields are appropriately reoriented after spatial normalization. Violating any one of these two assumptions, inter-individual averaging could reduce the accuracy of a fiber tracking algorithm, by blurring data that are poorly registered or properly aligned. We have used HAMMER algorithm, which has been shown to have very high accuracy[14].

6. REFERENCES

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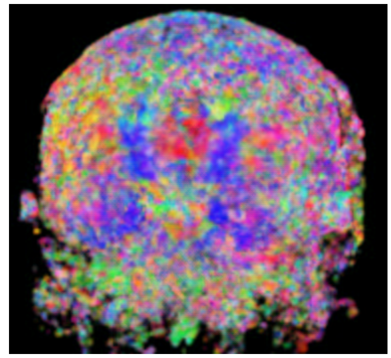

Fig. 2. *Experiment with real case: 3D volume rendering colormap of one individual DT brain. (red for X axis component, green for Y and blue for Z).*

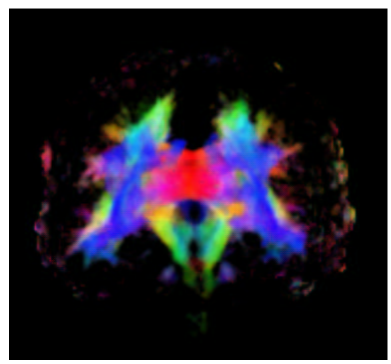

Fig. 3. *3D volume rendering colormap of the average result of 9 subjects.*

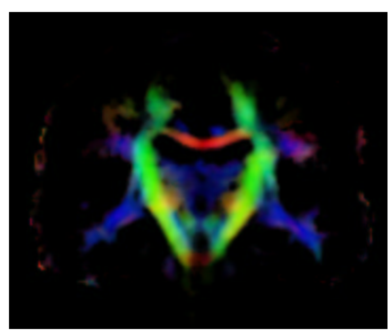

Fig. 4. *One typical coronal slice of the average result.*