Multi-modal Image Registration by Quantitative-Qualitative Measure of Mutual Information (Q-MI)*

Hongxia Luan[1], Feihu Qi[1], and Dinggang Shen[2]

[1] Department of Computer Science and Engineering,
Shanghai Jiao Tong University, Shanghai, China 200030
{luan-hx, fhqi}@cs.sjtu.edu.cn
[2] Section of Biomedical Image Analysis, Department of Radiology,
University of Pennsylvania, Philadelphia, PA 19104
dinggang.shen@uphs.upenn.edu

Abstract. This paper presents a novel measure of image similarity, called quantitative-qualitative measure of mutual information (Q-MI), for multi-modal image registration. Conventional information measure, i.e., Shannon's entropy, is a quantitative measure of information, since it only considers probabilities, not utilities of events. Actually, each event has its own utility to the fulfillment of the underlying goal, which can be independent of its probability of occurrence. Therefore, it is important to consider both quantitative and qualitative (i.e., utility) information simultaneously for image registration. To achieve this, salient voxels such as white matter (WM) voxels near to brain cortex will be assigned higher utilities than the WM voxels inside the large WM regions, according to the regional saliency values calculated from scale-space map of brain image. Thus, voxels with higher utilities will contribute more in measuring the mutual information of two images under registration. We use this novel measure of mutual information (Q-MI) for registration of multi-modality brain images, and find that the successful rate of our registration method is much higher than that of conventional mutual information registration method.

1 Introduction

Multi-modality image registration is important to accumulate information from different modality images for diagnosis of diseases, and to align preoperative images with intraoperative images for surgical planning. Mutual information has been suceessfully used as a measure of image similarity for both mono- and multi-modality image registration [1-3]. However, mutual information measurement only considers the matching of intensities, and ignores spatial information in the images. It should be noted that intensity matching not necessarily means anatomical matching. Therefore, it is important to design the registration methods that ensure the anatomical matching.

Spatial information, i.e., relationship of intensities between neighboring voxels, has been widely studied and incorporated into the mutual inforomation based registration

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procedure. In particular, many methods have been developed to include the image gradients into the registration [4,5]. In [5], the image registration is completed by maximizing both mutual information and matching of gradient maps between two images. Recently, distance map, calculated from grantient map, is proposed for multi-modality image registration [6], in order to make the registration method have larger capture ranges, since gradients are locally defined. On the other hand, the second-order information measures, i.e., the probabilities of co-occurence of intensity pairs within a certain size of image neighbourhood, were introduced into the mutual information based image registration [7]. Also, in order to employ multi-level spatial information simultaneously for image registration, Holden *et al* [8] firstly extracted features such as the luminance, the first and the second order derivatives of the scale space expansion of the image, and then registered two images by maximizing the multi-dimensional mutual information of the corresponding features.

It is important to note that most mutual information based registration methods treat each voxel equally during the registration procedure, regardless of whether some voxels are more useful than others in registration. Actually, different voxels even having same intensity should be treated differently in the registration procedure [9], according to their regional saliency values calculated from the scale-space map [10,11]. For example, WM voxels near to cortex should contribute more than WM voxels inside the large WM regions in measuring the mutual information between two images under registration. In this way, the registration algorithm can focus more on the registration of salient regions.

In this paper, we define a novel measure of image similarity, i.e., quantitative-qualitative measure of mutual information (Q-MI), for robust multimodality image registration. This new measure not only considers the probability of each image intensity, but also considers the utility of each image intensity, during the registration process. Here, we use the saliency value [10] as the utility for voxels in the image. By integrating both probability and utility into the definition of similarity of two images, our method has a much higher successful rate, compared to the conventional mutual information based registration methods, on the images obtained from the BrainWeb [12], indicating the robustness of our method to transformations.

2 Quantitative-Qualitative Measure of Mutual Information (Q-MI)

Before defining quantitative-qualitative measure of mutual information (Q-MI), we will briefly describe the basic concepts of information measure (2.1) and mutual information (2.2). Based on these definitions, we will give our definitions for quantitative-qualitative measure of information (2.3), and for quantitative-qualitative measure of mutual information (2.4).

2.1 Measure of Information

Information measure is a number related to the uncertainty or probability of occurrence of an event or outcome that conveys information [13]. Given a single event E_i with probability of occurrence p_i, the self-information of this event is defined as

$$H(E_i) = -\log p_i \tag{1}$$

The choice of a logarithmic base corresponds to the choice of a unit for measuring information. If the base 2 is used, the resulting units may be called binary digits, or more briefly bits [14]. The average information of a set of n events $E=(E_1, E_2, \ldots, E_n)$, each with probability of occurrence p_i, is defined as

$$H(E) = H(E_1, \ldots, E_n) = \sum_{i=1}^{n} p_i(-\log p_i) \tag{2}$$

The above is also called Shannon's entropy [14]. It weights the information per outcome by the probability of that outcome occurring. The Shannon's entropy is a measure of the amount of information required on the average to describe a set of events.

Let $P=(p_1, p_2, \ldots, p_n)$ be a finite discrete probability distribution of a set of n events $E=(E_1, E_2, \ldots, E_n)$ on the basis of an experiment whose predicted probability distribution is $Q=(q_1, q_2, \ldots, q_n)$. Then, a measure of directed divergence, which is called relative information or Kullback-Leibler distance [13], is defined as

$$D(E) = \sum_{i=1}^{n} p_i(\log \frac{p_i}{q_i}) \tag{3}$$

2.2 Measure of Mutual Information

Mutual information is an important concept in information theory and is defined according to relative entropy. Suppose we have two sets of events, $E=(E_1, E_2, \ldots, E_n)$ with probability distribution $P=(p_1, p_2, \ldots, p_n)$, and $F=(F_1, F_2, \ldots, F_m)$ with probability distribution $Q=(q_1, q_2, \ldots, q_m)$. Mutual information is a measure of the amount of information that the set of events E contains about the set of events F. Thus, mutual information is defined as the relative entropy between the joint distribution and the product distribution as follows,

$$MI(E,F) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(E_i, F_j) \log \frac{p(E_i, F_j)}{p_i q_j} \tag{4}$$

2.3 Quantitative-Qualitative Measure of Information

Shannon's entropy is a quantitative measure of information, since it considers all the events as random abstract events and neglects the particular aspects of those events. From the view of cybernetic system, Belis and Guiasu [15] presented a quantitative-qualitative measure of information in 1968. They thought that the occurrence of an event removes a double uncertainty, i.e., the quantitative one related to its probability of occurrence, and the qualitative one related to its utility for the fulfillment of the goal. The utility of an event is a subjective notion, and it is directly connected to the goal to achieve. Here, we emphasized that the utility of an event is independent of its objective probability of occurrence. For instance, an event of a small probability can have great utility, while an event of a great probability can have small utility.

Let $E=(E_1, E_2, \ldots, E_n)$ be a finite set of events representing the possible realizations of some experiments. Let $P=(p_1, p_2, \ldots, p_n)$ be the probabilities of occurrence of those events, and $U=(u_1, u_2, \ldots, u_n)$ be the utilities of those events. The quantitative-qualitative measure of information is

$$QH(E;U) = \sum_{i=1}^{n} u_i p_i (-\log p_i) \tag{5}$$

If all utilities are same, equation (5) becomes equation (2), which is Shannon's entropy.

Based on the work of Belis and Guiasu [15], Taneja [16] presented a quantitative-qualitative measure of relative information as follows,

$$QD(E;U) = \sum_{i=1}^{n} u_i p_i \log \frac{p_i}{q_i} \tag{6}$$

The quantity $u_i \log p_i$ is usually referred as *useful* self-information conveyed by an event with probability of occurrence p_i and utility u_i. Thus, the term $u_i \log(p_i/q_i) = u_i \log p_i - u_i \log q_i$ can be regarded as *useful* information gain in predicting the event E_i. When utilities in equation (6) are ignored, equation (6) becomes equation (3).

2.4 Quantitative-Qualitative Measure of Mutual Information (Q-MI)

As we mentioned above, Shannon's entropy-based mutual information only pays attention to the occurrence of events, and does not consider the particular aspects of events with respect to the goal. In order to consider the particular aspects of events, we define the quantitative-qualitative measure of mutual information (Q-MI) as follows, according to the quantitative-qualitative measure of relative information in equation (6),

$$QMI(E,F;U) = \sum_{i=1}^{n} \sum_{j=1}^{m} u(E_i, F_j) p(E_i, F_j) \log \frac{p(E_i, F_j)}{p_i q_j} \tag{7}$$

Like definitions given in equations (5) and (6), Q-MI focuses on the *useful* information that one set of events tells about another set of events. When utilities are the same, equation (7) becomes the definition of conventional mutual information.

3 Implementation

This section first describes the method of computing saliency values for each image location and using it as utility for that location. Then, the method of estimating utility for each intensity pair of two images is provided. Finally, the optimization method used in our registration algorithm is briefly described.

3.1 Saliency Measure

Each voxel in the image is unique, and it has its own roles. The only difference between those roles is the amount of significance. For example, the voxels that lie in the region of interest or at the boundary of region of interest are more significant for image analysis and understanding task, compared to the voxels that lie in the background. However, how to characterize each voxel still remains a hot topic in computer vision and pattern recognition fields.

Gradient operator is a simple image detector, able to identify the location of intensity changes. As we indicated in introduction, gradient map has been widely incorporated into the mutual information based registration methods. However, gradient is a local feature, and it is sensitive to noise. On the contrary, saliency measure [10], defined from scale-space map for each voxel in the image, is robust to noise and considers regional information. Accordingly, the saliency definition is adopted here for representing the significance of each voxel, and also the utility of this voxel in image registration.

Saliency measure is defined for each voxel in an image, and it is determined by analyzing entropy in the local regions of different size. For each voxel x, we first calculate the probability distribution of intensity i, $p_i(s,x)$, in a spherical region of radius s, centered at x. Then, we calculate the local entropy $L(s,x)$ from $p_i(s,x)$, as defined below,

$$L(s,x) = -\sum_i p_i(s,x)\log p_i(s,x) \tag{8}$$

The best scale s_x for the region centered at voxel x is selected as the one that maximizes local entropy $L(s,x)$. Since large scale and high local image difference are preferred, the saliency value of voxel x, $A(s_x,x)$, is defined by that maximal local entropy value, weighted by both the best scale s_x and a differential self-similarity measure in the scale space,

$$A(s_x,x) = L(s_x,x)\cdot s_x \cdot \sum_i \left\| \frac{\partial p_i(s,x)}{\partial s}\Big|_{s_x} \right\| \tag{9}$$

By measuring saliency over the whole image, each voxel has a saliency value to represent its significance in the image and also utility in image registration.

3.2 Calculating the Utility of Each Intensity Pair of Two Images

Once the saliency/utility has been defined for each voxel in the two images under registration, we are ready to define the utility for each intensity pair in the two images. Let $I_R(x)$ be the intensity of reference image R at location x, and $I_F(y)$ be the intensity of floating image F at location y. Similarly, let $A_R(x)$ and $A_F(y)$ be the saliency values of voxel x in R and voxel y in F, respectively. Then, the utility of an intensity pair (i,j) can be defined as,

$$u(i,j) = \sum_{\substack{x,y \in \Omega \\ I_R(x)=i,\ I_F(y)=j}} A_R(x) \cdot A_F(y) \tag{10}$$

where Ω is the overlap region of images R and F. In this paper, we use a *multiplication* operation to combine the saliency values from images R and F. Other combinations of utilities will be tested extensively in future.

3.3 Optimization

The registration problem based on Q-MI can be formulated as an optimization problem. Given a transformation T that maps a floating image F to match with a reference image R, we need to find an optimal transformation T^*,

$$T^* = \arg\max_T QMI(R, T(F); U) \tag{11}$$

such that our defined Q-MI, i.e., *QMI* in equation (11), is maximized.

Powell's multidimensional set method [17] is used to iteratively search for the maximum value of Q-MI, along each parameter via Brents's method [17]. To increase the robustness and also save the computation time, a multi-resolution framework of registration is performed.

4 Results

A number of experiments have been performed to demonstrate the performance of the proposed Q-MI in multi-modality image registration. The first set of experiments is used to test the performance of our method with respect to noise. The second set of experiments is used to evaluate the robustness of our proposed method. All experiments are performed on PD and T1 MR brain images obtained from Brain Web [12], and 2D slice images are used.

4.1 Visual Demonstration of the Robustness of Our Method to Noises

In this experiment, we evaluate our registration method on 2-dimensional PD and T1 MR brain images. The size of 2D image is 217x181, and the noise level is 1%, 5% and 9%, respectively. PD MR brain image is used as reference image, as displayed in Fig 1a, and T1 MR brain image is used as floating image, as displayed in Fig 1b. Figs 1c and 1d show the color-coded saliency measures for PD and T1 MR brain images, respectively. Since these are the simulation images, the different modality images have been aligned. To visually evaluate the performance of our registration algorithm with respect to different levels of noise, we plot the changes of Q-MI with respect to rotations around z-axis, horizontal shifting and vertical shifting, respectively. Here, parameters R_z, T_x, and T_y denote rotation around z-axis, horizontal shifting, and vertical shifting, respectively.

Figs 2a, 2b and 2c show the results of our Q-MI. It can be clearly observed that the curves of our Q-MI are smooth even in the case of large noise level, indicating the robustness of our method to noise.

4.2 Comparison on Robustness of Registration Methods

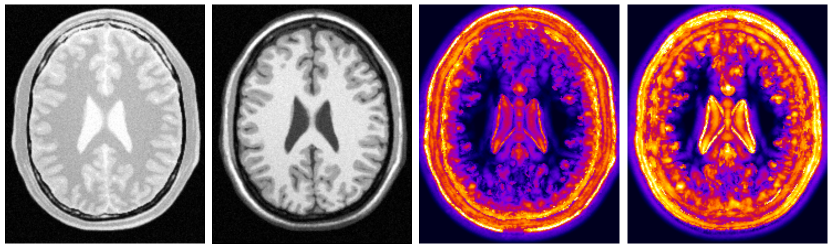
For validating the robustness and accuracy of the proposed method, we designed a series of controlled experiments on a pair of brain images with known transformations and noise levels. Similar to the experiments in 4.1, the PD MR brain image is used as the reference image, and the T1 MR brain image is used as the floating image. The floating image is transformed similarly in each of three datasets described below, but additive noises in both reference and floating images are different in the three datasets.

(1) *Test dataset 1*: 1% Gaussian noises were added in both reference and floating images. The floating image is simultaneously rotated and shifted, by a rotation angle uniformly sampled from the range of [-20, 20] degree and the horizontal and vertical shiftings (T_x and T_y) both uniformly sampled from the range of [-40mm, +40mm];
(2) *Test dataset 2*: 5% Gaussian noises were added in both reference and floating images. The floating image is similarly transformed as *Test dataset 1*;
(3) *Test dataset 3*: 9% Gaussian noises were added in both reference and floating images. The floating image is similarly transformed as *Test dataset 1*.

The test datasets 1~3 each generate 1000 randomly transformed images, therefore there are totally 3000 transformed floating images. We applied the registration algorithms, respectively based on conventional mutual information and our Q-MI, to independently register the reference image with each transformed floating image. In order to compare their performances fairly, we used the same optimization technique to perform registrations.

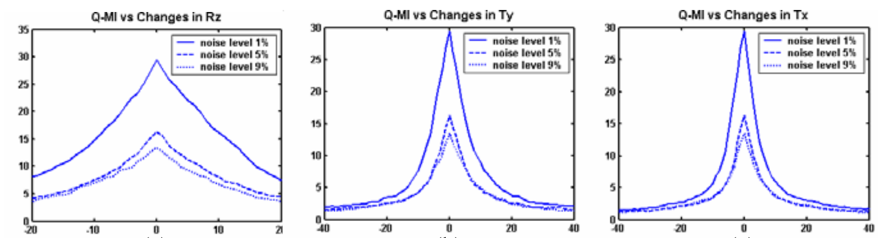
The registration result is regarded as successful if the differences between estimated transformations and ground-truth transformations are less than pre-defined thresholds, i.e., 2 degree for rotation and 2mm for shifting, which is similarly used in [18]. The successful rates of registration based on those two measures are listed in Table 1. Also, for those successful cases of registration, means and standard deviations of rotation errors and shifting errors are calculated, respectively, and listed in Table 2.

Based on Table 1, we can conclude that Q-MI based registration method has a higher success rate than conventional mutual information based method for each test dataset. That is, Q-MI based registration method is more robust, since utility has been incorporated into the mutual information definition. Based on Table 2 that compares registration accuracy for the successful cases, we can observe that the accuracy of Q-MI based registration is comparable to that of conventional mutual information based method.



(a) (b) (c) (d)

Fig. 1. The testing MR brain images. (a) PD MR brain image, (b) T1 MR brain image, (c,d) their color-coded saliency measures.



(a) (b) (c)

Fig. 2. Changes of Q-MI, with respective to rotations around z-axis R_z (a), horizontal shifting T_x (b), and vertical shifting T_y (c).

Table 1. Successful rates of two registration methods, based on Q-MI and conventional mutual information, respectively, in three datasets

Test Dataset	Successful rate	
	Mutual Information	Q-MI
Dataset 1	60.4%	84.9%
Dataset 2	54.6%	82.4%
Dataset 3	51.0%	76.5%

Table 2. Means and standard deviations of registration errors for the successful cases in three datasets, respectively

Test dataset	Mean and standard deviation					
	Mutual information			Q-MI		
	θ	T_x	T_y	θ	T_x	T_y
Dataset 1	0.03±0.04	0.35±0.53	0.41±0.62	0.13±0.11	0.41±0.59	0.51±0.73
Dataset 2	0.16±0.15	0.39±0.58	0.48±0.67	0.08±0.10	0.47±0.67	0.56±0.78
Dataset 3	0.24±0.24	0.42±0.61	0.50±0.70	0.15±0.17	0.55±0.75	0.58±0.79

5 Conclusion

We have presented a novel measure of image similarity, called quantitative-qualitative measure of mutual information (Q-MI), for registration of multi-modality images. This new measure integrates not only information obtained from the probability of intensity distribution, but also information obtained from the utility of each voxel, defined as value of saliency. This is different from other mutual information registration methods that use spatial information, since spatial information was not used as guidance for the calculation of mutual information. Our experiments on multi-modality MR brain images shows that the successful rate of our method is much higher than that of conventional mutual information based registration method, indicating the robustness of our method. In future, we will test our method extensively on other modality images of brains or other organs. We will also test on 3D images.

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