

and provides good prospects for practical applications of optimal bit allocation.

Table 1: Average number of iterations (ANI) and computation complexity (CC) required for optimal bit allocation in proposed and previous fast algorithms

Sequence	BFOS [2]		Shoham's [3]		Ramchandran's [4]		Proposed algorithm	
	ANI	CC	ANI	CC	ANI	CC	ANI	CC
Cyclegirl	1010.9	215.1	767.7	194.7	18.9	4.9	6.8	1.0
Table tennis	725.6	147.5	339.2	76.3	18.4	5.4	6.7	1.0
Flower garden	773.6	172.1	1427.1	336.0	17.7	4.1	6.3	1.0
Mobile and calendar	746.2	153.7	1674.9	350.3	17.6	3.9	6.8	1.0

CC represents the CPU time for bit allocation which is normalised with that of the proposed algorithm

Conclusions: In this Letter, we propose a new fast algorithm for optimal bit allocation. The algorithm is based on the bi-directional prediction scheme using the rate-distortion function and provides a substantial reduction in computational complexity. Our fast algorithm is useful for video coding applications such as VOD, CD-ROM etc., where the encoding delay is not important.

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Optimal axes for defining the orientations of shapes

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Indexing terms: Image processing, Pattern recognition

The authors present the concept of optimal axes for expressing the orientations of shapes, based on the first and second nonzero generalised complex moments. Compared with other reported methods, the technique gives the optimal (minimal number) axes needed to represent shape orientations.

Introduction: Many techniques exist for defining the orientations of 2D patterns, such as principal axes [1], shape matrices [2], mirror-symmetry axes [3-5], the line through the centroid and radius weighted mean [6], generalised principal axes (GPA) [7], fold principal axes (FPA) [8], and fold-invariant shape-specific points (FISSP) [9]. However, none of these methods is universal, i.e. each method can handle only certain shapes. To resolve this, Lin [10] developed a convenient tool to define universal axes of shape. The advantage of this method is that no preprocessing is required to

judge whether the input pattern is a rotational symmetrical shape (RSS), but the disadvantage is that it uses an expensive number of universal axes to represent the orientations of shape. Here we show that the orientations of an RSS, or non-RSS, can be represented by an optimal number of axes.

Universal property of shape function: Suppose that the original point of the co-ordinate has been assumed to be at the centre of the shape and $f(r, \theta)$ is the corresponding function of shape $f(x, y)$ in polar co-ordinates. Any kind of shape function $f(r, \theta)$ can be expressed by the Fourier expansion

$$f(r, \theta) = \sum_{m=-\infty}^{\infty} f_m(r) e^{-jm\theta}$$

where

$$f_m(r) = \frac{1}{2\pi} \int_0^{2\pi} f(r, \theta) e^{jm\theta} d\theta$$

If the given shape is an RSS with K folds (K -RSS), then its corresponding form of Fourier expansion is

$$f(r, \theta) = \sum_{l=-\infty}^{\infty} f_{K \times l}(r) e^{-j(K \times l)\theta}$$

where

$$f_{K \times l}(r) = \frac{K}{2\pi} \int_0^{2\pi/K} f(r, \theta) e^{j(K \times l)\theta} d\theta$$

Extracting optimal axes by GC moments: The pq -order generalised complex, (GC) moment of shape function $f(r, \theta)$ is defined as follows:

$$GC_{pq} = R_{pq} e^{j\varphi_{pq}} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} f(r, \theta) (r^p e^{jq\theta}) r dr d\theta$$

where p is a non-negative integer, and q a positive integer. Based on the general property of a shape function, we can prove that the condition for making GC_{pq} nonzero is $\int_0^{\infty} f(r) r^p dr \neq 0$. For a K -RSS, $f_q(r)$ must be zero when q is not a multiple of K .

Lemma 1, which was presented in [11], tells us that for certain P , if the first nonzero GC moment is GC_{p_1} , then the orientation of the shape is unique. Otherwise, more than half a line is needed to express the orientations of the shape. To reduce the number of half lines, we present lemmas 2 and 3. Lemma 2 indicates the possibility that if the number $(xq_1 + yq_2)$ is less than q_1 , then the number of half lines will be reduced to $(xq_1 + yq_2)$. The best choice is $xq_1 + yq_2 = 1$ which means that only a single half line is needed to represent the shape. Lemma 3 demonstrates the method to obtain appropriate values of x and y .

Lemma 1: If $GC_{pq} \neq 0$, then q half lines exist starting from the origin O and having directional angles

$$\theta_i = \frac{\varphi_{pq} + (i-1) \times 2\pi}{q} \quad \text{with } i = 1, 2, \dots, q$$

which are invariant to the translation, scaling and rotation of the identical shape.

Lemma 2: Let $GC_{p_1 q_1}$ and $GC_{p_2 q_2}$ be the first and second nonzero GC moments encountered in the sequence GC_{pq} with $q = 1, 2, \dots$, respectively. The phase of the combined moment $(GC_{p_1 q_1})^x (GC_{p_2 q_2})^y$ is $(x\varphi_{p_1 q_1} + y\varphi_{p_2 q_2})$. Then there are $(xq_1 + yq_2)$ optimal axes starting from the origin O and having directional angles

$$\theta_i = \frac{(x\varphi_{p_1 q_1} + y\varphi_{p_2 q_2}) + (i-1) \times 2\pi}{xq_1 + yq_2} \quad \text{with } i = 1, 2, \dots, (xq_1 + yq_2)$$

which are invariant to the translation, rotation and scaling of the identical image.

Table 1: (x, y) and $(xq_1 + yq_2)$ obtained by solving the linear programming problem

q1\q2	3	4	5	6	7	8	9
2	(-1,1)1	(1,0)2	(-2,1)1	(1,0)2	(-3,1)1	(1,0)2	(-4,1)1
3		(-1,1)1	(2,-1)1	(1,0)3	(-2,1)1	(3,-1)1	(1,0)3
4			(-1,1)1	(-1,1)2	(2,-1)1	(1,0)4	(-2,1)1
5				(-1,1)1	(3,-2)1	(-3,2)1	(2,-1)1
6					(-1,1)1	(-1,1)2	(-1,1)3

Lemma 3: There exist appropriate x and y making $1 \leq (xq_1 + yq_2) \leq q_1$. The appropriate values of x and y can be obtained by solving the linear programming problem and selecting the smaller $|x|$ and $|y|$ in the solution set. The objective function is $\min_{x,y}(xq_1 + yq_2)$ and the constraint condition is $xq_1 + yq_2 \geq 1$.

According to Lemma 3, we can obtain Table 1 which gives the appropriate values of x and y for some q_1 and q_2 . In Table 1, the first column contains possible q_1 and the first row contains possible q_2 . The other cells correspond to the solutions, (x,y) and $xq_1 + yq_2$, for different q_1 and q_2 .

Using alternating energy to control orientation detection: Since the order p in GC_{pq} is always fixed when GC_{pq} is applied in defining shape orientation, a selection rule is proposed for choosing a suitable p based on the alternating energy of the Fourier spectrum of the shape.

Suppose that $h_p(\theta)$ is a 1D function obtained from $f(r,\theta)$, $h_p(\theta) = \int_0^{\infty} f(r,\theta)r^p dr$. The Fourier transform of $h_p(\theta)$ is $H_p(q) = 1/2\pi \int_{-\pi}^{\pi} h_p(\theta)e^{iq\theta} d\theta$, which leads to $\int_0^{2\pi} [h_p(\theta)]^2 d\theta = 2\pi \sum_{q=-\infty}^{\infty} |H_p(q)|^2$. Notice that $GC_{pq} = H_p(q)$, which implies that GC_{pq} can be directly obtained from the Fourier transform of the 1D function $h_p(\theta)$ when the order p has been fixed. The ratio

$$a_p = 1 - \frac{2\pi \|H_p(0)\|^2}{\int_0^{2\pi} [h_p(\theta)]^2 d\theta}$$

can then be used to determine whether the 1D function $h_p(\theta)$ has a strong periodicity. In our study, we select a 1D function whose corresponding a_p is $> 5\%$.

Besides applying the function energy to determine the order p , function energy is also suggested here to determine whether the remaining nonzero GC moments can be detected, or not. Let

$$b_q = 1 - \frac{2\pi \left(\|H_P(0)\|^2 + 2 \sum_{m=1}^q \|H_P(m)\|^2 \right)}{\int_0^{2\pi} [h_P(\theta)]^2 d\theta}$$

If b_q is small, it means that little alternating energy is left, and no more nonzero GC moments should be detected. In our study, the threshold for b_q has been set to 1%.

Algorithm: Our technique is summarised as follows:

- (i) Translate the co-ordinate system so that the origin O becomes the centroid of the given shape.
- (ii) Determine the parameter p in the GC moment GC_{pq} based on the selection criteria that a 1D data sequence should have enough alternating energy.
- (iii) Compute the first nonzero GC moments of $f(x,y)$, GC_{pq_1} . If $q_1 = 1$, then the orientation of the shape is uniquely defined by the axis starting from the origin O and having directional angle φ_{pq_1} . Otherwise, go to (iv).
- (iv) Compute the second nonzero GC moments of $f(x,y)$, GC_{pq_2} . The orientations of the shape are defined by $N = x_1q_1 + y_1q_2$ optimal axes starting from the origin O and having directional angles

$$\theta_i = \frac{(x_1\varphi_{Pq_1} + y_1\varphi_{Pq_2}) + (i-1) \times 2\pi}{N} \quad i = 1, \dots, N$$

Experiments: In our experiments, the orientations of the shape are determined by the method of universal axes reported in [10] and our method of optimal axes represented here.

Fig. 1 shows a non-RSS, the first and second nonzero GC moments of which are $GC_{0,3}$ and $GC_{0,4}$ separately. Thus, three uni-

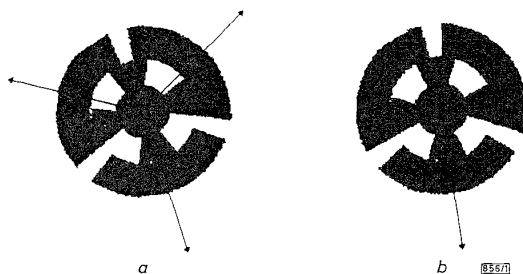


Fig. 1 Non-RSS
First and second nonzero GC moments are $GC_{0,3}$ and $GC_{0,4}$
a Universal axes
b Optimal axes

versal axes are needed to express the orientations of this shape, which is shown in Fig. 1a. However, only a single optimal axis is needed to define the shape orientation uniquely, and the directional angle of this optimal axis is $(GC_{0,4} - GC_{0,3})$. That is, for almost every kind of non-RSS, there exists a single unique optimal axis.

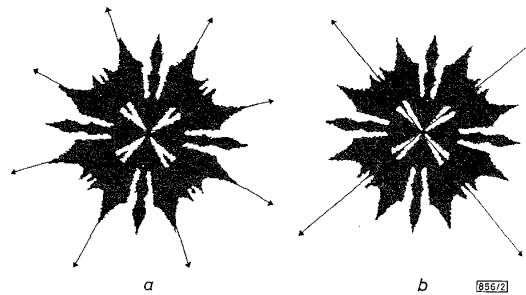


Fig. 2 4-RSS
First and second nonzero GC moments are $GC_{0,8}$ and $GC_{0,12}$
a Universal axes
b Optimal axes

The original image of Fig. 2 was obtained from Fig. 6 shown in [9]. This is a 4-RSS, whose first and second nonzero GC moments are $GC_{0,8}$ and $GC_{0,12}$, respectively. There are eight universal axes which are shown in Fig. 2a. However, there are four optimal axes, which is equal to the fold number of this RSS. The result is shown in Fig. 2b. It shows that for almost every kind of RSS, the number of optimal axes is equal to the fold number of the RSS.

Conclusions: We present a technique to extract optimal axes for almost every kind of shape. Optimal axes are obtained by solving the linear programming problem. The number of these optimal axes is, for most cases, less than and, for some cases equal to, the number of universal axes. Only a single optimal axis is needed to express the orientation of a non-RSS, and for an RSS, the number of optimal axes is equal to the fold number of the shape.

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