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An efficient fuzzy algorithm for aligning shapes under affine transformations

Zhong Xue[a], Dinggang Shen[b], Eam Khwang Teoh[a,*]

[a]School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore
[b]Department of Radiology, School of Medicine, Johns Hopkins University, Baltimore, MD 21287, USA

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Abstract

A fuzzy algorithm for aligning object shapes under affine transformations is proposed in this paper. The algorithm, with the name of fuzzy alignment algorithm (FAA), extends Marques' algorithm to affine transformations. It can efficiently estimate the point correspondence and the relevant affine transformational parameters between the feature points of the object shape and the reference shape. In this algorithm, the fuzzy point-correspondence degrees are used to describe an uncertainty point assignment, then both the parameters of the affine transformation and the fuzzy correspondence degrees are iteratively calculated by minimizing a constrained fuzzy objective function. To prevent FAA from sinking into local minimum when the shapes are greatly deformed, an initialization method based on affine invariants is designed. Comparing to the eigenvector method, the effectiveness and robustness of the proposed algorithm is investigated with a sensitivity study based on randomly generated points. At last, good performance of FAA is illustrated with several experiments on aligning digits and object shapes.

Keywords: Affine invariant; Affine transformation; Correspondence; Fuzzy clustering; Pose estimation; Shape alignment

1. Introduction

The alignment of a pair of shapes, or estimation of the point correspondence and the relevant transformational parameters between the feature points of two shapes, has attracted much attention in the areas of pattern recognition and computer vision for decades. Different strategies have been developed in literature, namely, estimating the transformational parameters by least-squares algorithms [1,2], calculating the point correspondence by proximity matrices [3,4], pose clustering and estimation approach [5,6], template matching methods [7,8] and accumulation of evidence [9,10], etc.

In Ref. [1], Umeyama proposed a least-squares (LS) method to compute the similarity transformational parameters between two point patterns. The algorithm, which is proved to be a strict solution to the point matching problem, can give the correct parameters even when the data is corrupted. Werman and Weinshall [2] defined affine-invariant expressions for measuring the distance between 2D point sets, which can then be used to match the points between the two sets using the LS method. However, in both algorithms, the point correspondence between the two sets has to be known a priori.

To determine the point correspondence, Scott and Longuet-Higgins [3] proposed a proximity matrix strategy. The proximity matrix is an affinity measure based on the inter-set point distances. By applying singular value decomposition (SVD), a competition scheme was utilized to determine the correspondence according to the principle of proximity and exclusion. Experiments showed that good results can be achieved under 2D translations, expansions, and small shears, but when there is large rotation in the image plane, the algorithm cannot cope with it successfully [4]. To overcome this weakness,

* Corresponding author. Tel.: + 65-790-5393; fax: + 65-792-0415.
E-mail address: eekteoh@ntu.edu.sg (E.K. Teoh).

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Shapiro and Brady [4] developed an eigenvector approach, where the proximity matrices are created using the intra-set point distances instead of the inter-set distances, and the association matrix calculated from the feature vectors of the proximity matrices is utilized to determine the point correspondence. However, this algorithm can only deal with the transformations under small skews, and because the sign of each eigenvector is not unique, a time consumptive sign correction stage is required.

As stated in Ref. [11], estimation of the transformational parameters requires reasonable point correspondence, while good point correspondence can be acquired based on the transformational geometry information. Therefore, algorithms were developed to iteratively estimate the transformational parameters and refine correspondence matches [11–14]. In Ref. [11], Cross and Hancock proposed a dual-step expectation maximization (EM) algorithm for matching, which is based on the graphical structure and Bayesian estimation. Gold et al. [12] developed an objective function and used a combination of optimization techniques, including deterministic annealing and the softassign [15], to obtain the estimation of the correspondence and the affine transformational parameters iteratively. From the formation of its objective function, we can see that there are bound constraints to the scale and shear components of the affine transformation. Therefore, it is not suitable for estimating the affine transformations with large scales and large shears.

An alternative way to align the shapes is fuzzy algorithm. Since it is ambiguous to determine the point correspondence, the adopting of fuzzy concept appears to be a suitable method to deal with this problem. In Ref. [14], Marques proposed a fuzzy algorithm for the alignment of curves and surfaces using iterative estimation. In Marques' method, to align a pair of shapes or point sets, confidence degrees are utilized to describe each combination between the points in two shapes. Then, the rotation variables and the confidence degrees are obtained by minimizing a fuzzy objective function iteratively. The algorithm works quite well for aligning shapes under different rotations and translations. However, because the algorithm is designed to estimate Euclidean transformations, the performance deteriorates in the presence of skews and scaling, which greatly eliminates the practical use of the algorithm.

This paper extends Marques' fuzzy algorithm to affine transformations, which results in the so-called fuzzy alignment algorithm (FAA). Similar to Marques' algorithm, the fuzzy point-correspondence degree functions between each point in the first set and all the points in the second set are prescribed. An iterative method is proposed to estimate both the parameters of the affine transformation and the fuzzy point-correspondence degrees by minimizing a fuzzy objective function under the constraints: the columns of the fuzzy point-correspondence matrix must add up to one. Note that in Marques' algorithm, the transformational matrix A is limited to be unitary, while in FAA, the only constraint on A is nonsingular. The main advantage of FAA lies in that, it can obtain the good alignment results not only from the close and open curve boundaries, but also from the broken boundaries or other feature point sets of shapes under affine transformations.

If the initialization values of the fuzzy point correspondence are randomly selected, as a gradient method, FAA has probability to sink into local minimum, especially when the shapes are highly deformed or the number of point is large. Methods are developed to solve this problem, such as simulated annealing, genetic algorithm and various initialization methods. In this paper, we design an initialization method for FAA based on affine-invariant features. Experiments show that with the initialization, the probability of sinking into local minimum or some undesirable results is decreased.

In the experiments, the performance of FAA with initialization is compared with that of the traditional eigenvector method proposed by Shapiro and Brady [4], where randomly generated data and affine transformational parameters are used. The comparison is done through a sensitivity study, including the sensitivity of the algorithms to the number of points, random point measure error, missing and spurious object points. Based on the comparison, FAA is utilized to align the handwriting digits and object shapes in different conditions. The results demonstrate FAA's good performance for aligning shapes under affine transformations.

The paper is organized as follows. Section 2 formulates the problem and introduces FAA in detail. Section 3 presents the initialization method for FAA based on affine invariants. In Section 4, experiments on algorithm comparison, sensitivity analysis, and alignment applications are presented to demonstrate the good performance of the algorithm. At last in Section 5, a concluding remark is given.

2. Fuzzy alignment algorithm (FAA)

2.1. Problem formulation

Denote the two point sets to be aligned as observed (object) point set and reference point set, the alignment problem is to determine the affine transformation and the point correspondence between the two point sets. Mathematically, let X and Y be the reference point set and object point set, respectively, it is assumed that the points $\mathbf{y}_i \in Y$ ($\mathbf{y}_i \in R^q$, $1 \leqslant i \leqslant M$) and $\mathbf{x}_j \in X$ ($\mathbf{x}_j \in R^q$, $1 \leqslant j \leqslant N$) are related with the following affine transformation:

$$\mathbf{y}_i = T(\mathbf{x}_j) + \delta, \tag{1}$$

where T is an affine transformation operator. δ indicates the random noise and error due to imaging error, measure error, and noise. In this paper, we consider the 2-D alignment, where $q = 2$. The algorithm for 3-D case can be extended straightforward. The affine transformation is denoted as

$$T(\mathbf{x}_j) = A\mathbf{x}_j + \mathbf{t}, \tag{2}$$

where A is a 2×2 nonsingular matrix and $\mathbf{t}$ is a translation vector. If we know the point correspondence between X and Y, the affine transformational parameters can be directly estimated by the LS method. Unfortunately, in practice, it is difficult to determine which point in X corresponds to the point $\mathbf{y}_i$ in Y. One of the solutions is to introduce w_{ij}, a point-correspondence degree between $\mathbf{y}_i$ and $\mathbf{x}_j$, according to fuzzy techniques. The point-correspondence degree functions satisfy the following normalization condition:

$$\sum_j w_{ij} = 1, \qquad \forall i, \quad w_{ij} \in [0,1],\ 1 \leqslant i \leqslant M,\ 1 \leqslant j \leqslant N. \tag{3}$$

To estimate the affine transformational parameters between X and Y, the fuzzy objective function E is defined,

$$E(A,\mathbf{t},w) = \sum_{i,j} w_{ij}^p \|\mathbf{y}_i - (A\mathbf{x}_j + \mathbf{t})\|^2, \tag{4}$$

where $p > 1$ is a parameter to control the fuzziness of the solution, $w = [w_{ij}]$ is fuzzy point-correspondence degree, and $\|.\|$ refers to the Euclidean norm. The difference between Eq. (4) and the criterion of the LS method ($E' = \sum_i \|\mathbf{y}_i - (A\mathbf{x}_i + \mathbf{t})\|^2$) is that, in Eq. (4), a matrix w is used to describe the unknown point correspondence.

2.2. FAA for aligning shapes

FAA estimates the transformational parameters and the point correspondence degree functions by minimizing the fuzzy objective function, using the constraints in Eq. (3). The whole process can be divided into two iterative steps: (i) Estimate the transformational parameters $(\hat{A}, \hat{\mathbf{t}})$. (ii) Update the estimation of point-correspondence degree functions $\hat{w}_{ij}$.

2.2.1. Estimation of the transformational parameters

We first assume that the point-correspondence degree functions are known, and the problem is to estimate the parameters of the affine transformation, A and $\mathbf{t}$, by minimizing of the objective function in Eq. (4).

2.2.2. Estimation of t

To estimate the translation $\mathbf{t}$, which minimizes Eq. (4) when all the other parameters are given, the partial derivative method is utilized. A necessary condition of the minimization is

$$\left.\frac{\partial E(A,\mathbf{t},w)}{\partial \mathbf{t}}\right|_{\mathbf{t}=\hat{\mathbf{t}}} = 0,$$

$$\sum_{ij} w_{ij}^p(\mathbf{y}_i - A\mathbf{x}_j - \hat{\mathbf{t}}) = 0. \tag{5}$$

So the estimated translation is

$$\hat{\mathbf{t}} = \bar{\mathbf{y}} - A\bar{\mathbf{x}}, \tag{6}$$

where $\bar{\mathbf{x}} = 1/C\sum_{ij} w_{ij}^p \mathbf{x}_j$, $\bar{\mathbf{y}} = 1/C\sum_{ij} w_{ij}^p \mathbf{y}_i$, $C = \sum_{ij} w_{ij}^p$. $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ can be considered as the weighted mass centers of X and Y, respectively. $\hat{\mathbf{t}}$ is the estimated translation of the affine transformation.

2.2.3. Estimation of A

In Marques' method, A is an unitary matrix (rotation matrix), it can be calculated by SVD decomposition [14]. However, in this paper, except for nonsingular, there is no other constraint to the transformation matrix A. Substituting Eq. (6) into Eq. (4), we can get

$$\begin{aligned}
E(A,\hat{\mathbf{t}},w) &= \sum_{i,j} w_{ij}^p \|(\mathbf{y}_i - \bar{\mathbf{y}}) - A(\mathbf{x}_j - \bar{\mathbf{x}})\|^2 \\
&= \sum_{i,j} w_{ij}^p [(\mathbf{y}_i - \bar{\mathbf{y}}) - A(\mathbf{x}_j - \bar{\mathbf{x}})]^{\mathrm{T}} [(\mathbf{y}_i - \bar{\mathbf{y}}) \\
&\quad - A(\mathbf{x}_j - \bar{\mathbf{x}})] = \sum_{i,j} w_{ij}^p [(\mathbf{y}_i - \bar{\mathbf{y}})^{\mathrm{T}}(\mathbf{y}_i - \bar{\mathbf{y}}) \\
&\quad - 2(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}} A^{\mathrm{T}} (\mathbf{y}_i - \bar{\mathbf{y}}) + (\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}} A^{\mathrm{T}} A (\mathbf{x}_j - \bar{\mathbf{x}})].
\end{aligned} \tag{7}$$

A necessary condition for A to minimize the fuzzy objective function is

$$\left.\frac{\partial E(A,\hat{\mathbf{t}},w)}{\partial A}\right|_{A=\hat{A}} = 0,$$

that is,

$$-\sum_{ij} w_{ij}^p (\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}} + \hat{A}\sum_{ij} w_{ij}^p (\mathbf{x}_j - \bar{\mathbf{x}})(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}} = 0, \tag{8}$$

$$\hat{A} = \left[\sum_{ij} w_{ij}^p (\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}}\right]\left[\sum_{ij} w_{ij}^p (\mathbf{x}_j - \bar{\mathbf{x}})(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}}\right]^{-1}, \tag{9}$$

where $\sum_{ij} w_{ij}^p (\mathbf{x}_j - \bar{\mathbf{x}})(\mathbf{x}_j - \bar{\mathbf{x}})^{\mathrm{T}}$ is always nonsingular when there exist at least three points in X, which are not located in the same line.

2.2.4. Estimation of fuzzy point-correspondence degrees

The estimation of point-correspondence degree functions, $\hat{w}_{ij}$, can be accomplished by minimizing Eq. (4), plus using the constraints in Eq. (3). According to Lagrange method, necessary condition for the minimization can be derived from the following Lagrange function:

$$L = \sum_{i,j} w_{ij}^p \|\mathbf{y}_i - (\hat{A}\mathbf{x}_j + \hat{\mathbf{t}})\|^2 + \sum_i \lambda_i \left(\sum_j w_{ij} - 1 \right), \quad (10)$$

where λ_i are Lagrange multipliers. Let the partial derivatives of L with respect to w_{ij} equal to zero, we get the stationary conditions,

$$p\hat{w}_{ij}^{p-1} \|\mathbf{y}_i - \hat{A}\mathbf{x}_j - \hat{\mathbf{t}}\|^2 + \lambda_i = 0. \quad (11)$$

By using the above conditions and the normalization constraints, the Lagrange multipliers can be obtained as follows:

$$\lambda_i = -\left[\frac{1}{N\sum_{m=1}^{N} (1/p\|\mathbf{y}_i - \hat{A}\mathbf{x}_m - \hat{\mathbf{t}}\|^{1/(p-1)})} \right]^{p-1} \quad (12)$$

and from Eq. (11), the estimate of point-correspondence degree functions is

$$\hat{w}_{ij} = \frac{1}{\sum_m (\|\mathbf{y}_i - \hat{A}\mathbf{x}_j - \hat{\mathbf{t}}\|^{2/(p-1)} / \|\mathbf{y}_i - \hat{A}\mathbf{x}_m - \hat{\mathbf{t}}\|^{2/(p-1)})}, \quad (13)$$

where $\|\mathbf{y}_i - \hat{A}\mathbf{x}_m - \hat{\mathbf{t}}\|$ (for all i and m) are assumed to be non-zero.

The whole algorithm can be summarized as

Step 1: Initialization. Give the feature point sets of the reference (prototype) and the object shapes, X and Y, initialize the point-correspondence degree functions $\hat{w}_{ij}^{(1)}$. Set energy threshold ε, and let $k = 1$.

Step 2: Calculate $\hat{\mathbf{t}}^{(k)}$ according to Eq. (6) and the matrix $\hat{A}^{(k)}$ of affine transformation according to Eq. (9), by replacing w_{ij} with $\hat{w}_{ij}^{(k)}$.

Step 3: Estimate the point-correspondence degree functions, $\hat{w}_{ij}^{(k+1)}$, which will be used in the next iteration, according to Eq. (13) by setting $\hat{A}$ as $\hat{A}^{(k)}$ and $\hat{\mathbf{t}}$ as $\hat{\mathbf{t}}^{(k)}$.

Step 4: Calculate the fuzzy objective function $E^{(k)}$, by substituting $\hat{A}^{(k)}$, $\hat{\mathbf{t}}^{(k)}$ and $\hat{w}_{ij}^{(k)}$ into Eq. (4). If

$$|E^{(k)} - E^{(k-1)}| < \varepsilon, \quad k \geqslant 2$$

then the algorithm ends, and $\hat{A}$, $\hat{\mathbf{t}}$ and $\hat{w}$ are the results; else let $k = k + 1$ and go to step 2 to begin another iteration.

3. Initialization method for FAA

In this section, to prevent the algorithm from running into local minimum and make the algorithm more feasible, the initialization method for FAA based on affine invariants is designed. The idea for initializing FAA is to set the fuzzy-correspondence degrees according to the most possibly corresponded points, based on the affine-invariant features of the points to be aligned. In this way, local minimum of FAA can be prevented.

In this section, the well-known affine-invariant features, *the ratio of two areas*, are recruited to initialize FAA. Based on this fact, the interior area proportion of two polygons P_1 and P_2 is invariant,

$$S(P_1)/S(P_2) = S(T(P_1))/S(T(P_2)), \quad (14)$$

where $S(P)$ is the interior area of the polygon P, and $T(.)$ is an affine transformation operator.

The initialization method for FAA is presented and described as follows.

Firstly, the proximity matrices, H_X and H_Y, to record the area features of each point set are calculated by

$$H_{ij}^X = \mathrm{e}^{-r_{ij}^X/\sigma}, \quad H_{mn}^Y = \mathrm{e}^{-r_{mn}^Y/\sigma}, \quad (15)$$

where

$$r_{ij}^X = s_X(i,j)/S_X, \quad r_{mn}^Y = s_Y(m,n)/S_Y \quad (16)$$

and

$$s_X(i,j) = \left\| \begin{matrix} \mathbf{x}_i & \mathbf{x}_j & \bar{\mathbf{x}} \\ 1 & 1 & 1 \end{matrix} \right\|, \quad s_Y(m,n) = \left\| \begin{matrix} \mathbf{y}_m & \mathbf{y}_n & \bar{\mathbf{y}} \\ 1 & 1 & 1 \end{matrix} \right\|,$$
$$1 \leqslant i,j \leqslant N, \quad 1 \leqslant m,n \leqslant M, \quad (17)$$

where S_X and S_Y are the areas enclosed by the lines joining the outmost boundary points in X and Y, $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are the central points of X and Y, and σ is the scaling parameter. Therefore, the elements in the matrices H_X and H_Y are affine invariant.

Secondly, an association method is applied to find out the most possibly corresponded points from the proximity matrices H_X and H_Y quickly. The eigenvectors and eigenvalues of the matrices are calculated and rearranged so that the diagonal matrices D_X and D_Y contain the eigenvalues in decreasing absolute values.

$$H_X = V_X D_X V_X^{\mathrm{T}}, \quad H_Y = V_Y D_Y V_Y^{\mathrm{T}}, \quad (18)$$

where the columns of V_X and V_Y are the eigenvectors of H_X and H_Y, respectively. Since X and Y are the feature point sets of the same shape, the values of D_X and D_Y are close to each other. Therefore, let $F_{i,X}$ be the ith row of V_X, and $F_{j,Y}$ be the jth row of V_Y, the association between $F_{i,X}$ and $F_{j,Y}$ can give the possible point correspondence. However, because the sign of the eigenvectors may not be unique, if the signs of the eigenvectors in V_Y remain unchanged, the sign of each eigenvector in V_X should be found so that the association matrix of the row vectors stands for the point correspondence. This is a time consumptive combination optimization (CO) problem. The association matrix, Z, is defined by

$$Z_{ij} = \|F_{i,X} - F_{j,Y}\|^2, \quad 1 \leqslant i \leqslant N,\ 1 \leqslant j \leqslant M \quad (19)$$

the computing time can be greatly decreased by using a simple method that only consider the signs of the principal eigenvectors that have larger absolute eigenvalues. This is similar to the principal component analysis (PCA) method, and only the first L columns of V_X and V_Y are considered when calculating Z, and $L \ll M$, $L \ll N$. In this way, $F_{i,X}$ and $F_{j,Y}$ become the vectors with length of L. The reduction decreases the computing time since generally L is much smaller than M and N.

Finally, the most possibly corresponded C point pairs, $\Omega = \{(\mathbf{x}_i, \mathbf{y}_j) \mid \text{the two points are possibly corresponded}\}$, can be found according to the association matrix Z, and the fuzzy-correspondence degrees of FAA are initialized as follows:

$$w_{j,i} = \begin{cases} d_1 & \text{if } (\mathbf{x}_i, \mathbf{y}_j) \in \Omega, \\ d_2 & \text{otherwise,} \end{cases} \tag{20}$$

where $0 \leqslant d_2 \leqslant d_1 \leqslant 1$ is proper initialization value of w.

4. Experimental results

In this section, experiments are designed to evaluate the proposed algorithm. First, FAA is compared with the traditional eigenvector method based on a sensitivity study. By using randomly generated points and affine transformational parameters, the sensitivity of the two algorithms to the number of points, random point measure error, missing and spurious object points is investigated and compared. The second set of experiments focuses on aligning real object shapes using FAA, where handwriting digits and object shapes are used, the results demonstrate the effectiveness and robustness of the proposed method.

4.1. Algorithm comparison based on sensitivity study

To demonstrate the effectiveness of FAA with initialization algorithm, comparison experiments with the eigenvector method based on a sensitivity study are presented. The results have been obtained using random points and transformations, which is reasonable to evaluate the performance of the algorithms.

The reference points, $\mathbf{x} \in X$, are uniformly distributed in the unit box $[0,1] \times [0,1]$, and the transformational matrix A is given by

$$A = R_1 \Gamma R_2, \tag{21}$$

where R_1 and R_2 are rotation matrices with uniformly random angles, $\theta \in [-\pi, \pi]$, and

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$$

is the scaling matrix. Scales γ_1 and γ_2 are independent variables uniformly distributed in $[0,3]$. $\mathbf{t}$ is the translation vector, and it is randomly distributed in $[0,1] \times [0,1]$.

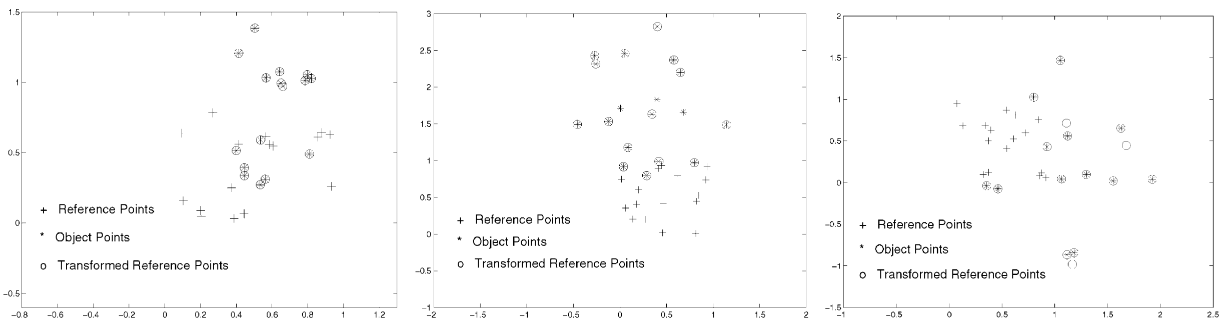
Then, the object points can be obtained by

$$\mathbf{y}_i = A\mathbf{x}_i + \mathbf{t} + \delta, \quad 1 \leqslant i \leqslant N. \tag{22}$$

The elements of Y are re-arranged randomly so that the point correspondence between X and Y is unknown. δ denotes noise vector, which describes the random point measure error.

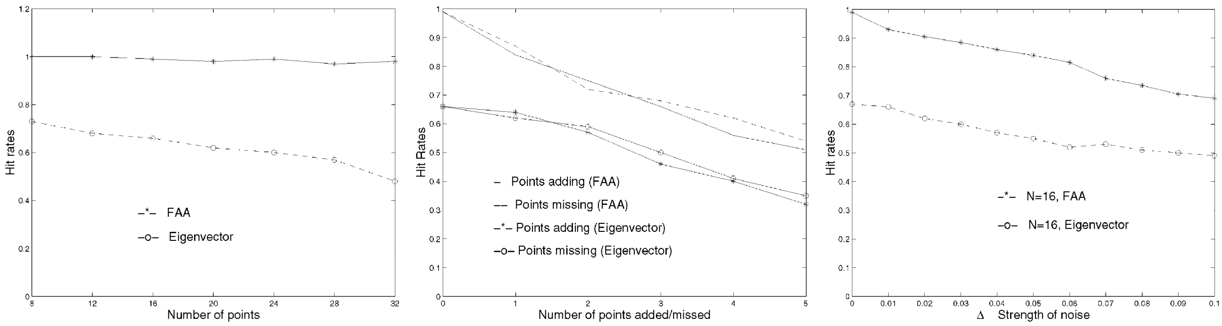
FAA and the eigenvector method are used to estimate the affine transformational parameters and the point correspondence between X and Y. Figs. 1 and 2 depict the comparison results. In Fig. 1, some example results of FAA are given, '+' represents the reference point, '*' indicates the object point, and '○' shows the transformed point of the reference using the estimated transformational parameters. In Fig. 1(a), the numbers of the points in X and Y are both 16. In Fig. 1(b), three spurious object points are inserted, and three object points are missed in Fig. 1(c).

To evaluate the performance of the algorithms quantitatively, hit rate is utilized in this paper. Hit rate is defined as the percentage of the correct alignment results among a given number of experiments. A threshold is set to gate the defined error, and an alignment result is said



(a) (b) (c)

Fig. 1. Examples of FAA to align randomly generated points. (a) $N = M = 16$, (b) three spurious object points are inserted, $N = 16$, $M = 19$, (c) three object points are missed, $N = 16$, $M = 13$.



(a) (b) (c)

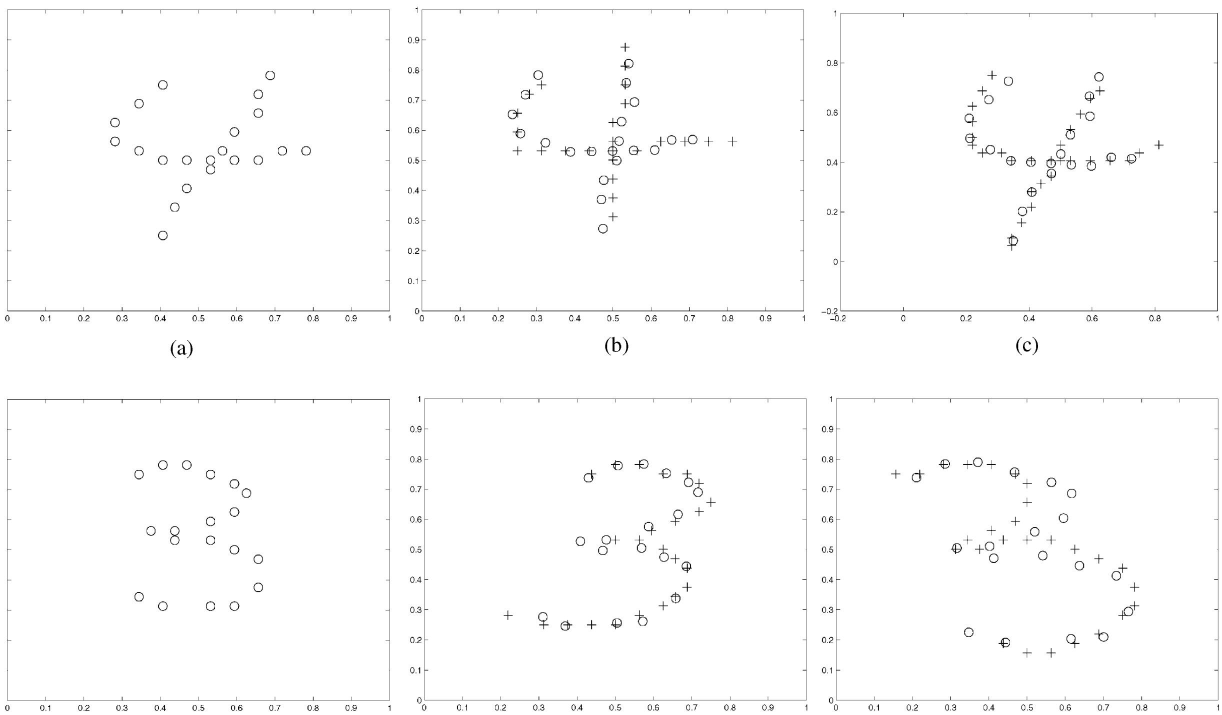
Fig. 2. Sensitivity of FAA to the number of points, the number of missing and spurious points, and point measure noise.

to be correct if the value of its error is smaller than the threshold (In the experiments, this threshold is set to 0.1). The error function is given by least-squares error between the object points and the transformed points of the reference by using the estimated transformational parameters and the real point correspondence.

$$e = \frac{1}{C}\sum \|A^{-1}(\mathbf{y}_i - \mathbf{t}) - \mathbf{x}_j\|^2, \tag{23}$$

where $\mathbf{x}_j$ and $\mathbf{y}_i$ are the C real corresponded point pairs between X and Y. Each hit rate given in the figures is calculated with respect to 200 tests.

In many real applications of alignment, the results might be affected by the number of the points aligned, point measure error, spurious or missing object points, etc. Therefore, the sensitivity of FAA and the eigenvector method is examined and compared.



(a) (b) (c)

(d) (e) (f)

Fig. 3. Alignment results of digits. (a) and (d), the reference points of digits. (b), (c), (e) and (f), object digit points, '+', and the alignment results, '○'.

Firstly, the sensitivity of both methods to the number of points is examined. Fig. 2(a) shows the alignment results of FAA and the eigenvector method, where the number of points to be aligned varies from 8 to 32 with a step of 4. From the figure, we see FAA can get very high hit rates (nearly one) for different number of points, while the hit rates of the eigenvector method are much lower and decrease faster when the number of points increases. This indicates that FAA performs well for large number of points under affine transformations.

Secondly, the sensitivity of both algorithms to spurious/missing points is studied. The examples of this case are shown in Fig. 1(b) and (c). The sensitivity curves are plotted in Fig. 2(b), where the point number of X is 16, and the number of inserted/missed points varies from 0 to 5. From the figure, we can see the hit rates of FAA decrease from 0.98 to 0.53 when there are about 1/3 of the object points inserted or missed, and those of the eigenvector method changes from 0.66 to 0.34.

Thirdly, the sensitivity to the random point measure error is investigated. Similarly, randomly generated points and affine transformational parameters are utilized, while the object points are mixt with random point measure error, δ. δ is uniformly distributed in the box $[-\Delta, \Delta] \times [-\Delta, \Delta]$, Δ is denoted as the strength of the noise. Fig. 2(c) is the result when $N = 16$, where δ changes from 0 to 0.1. The solid signature with stars in the figure is the hit rates of FAA, while the circled one is the result of the eigenvector method. The figure shows that the performance of both the algorithms decreases along with

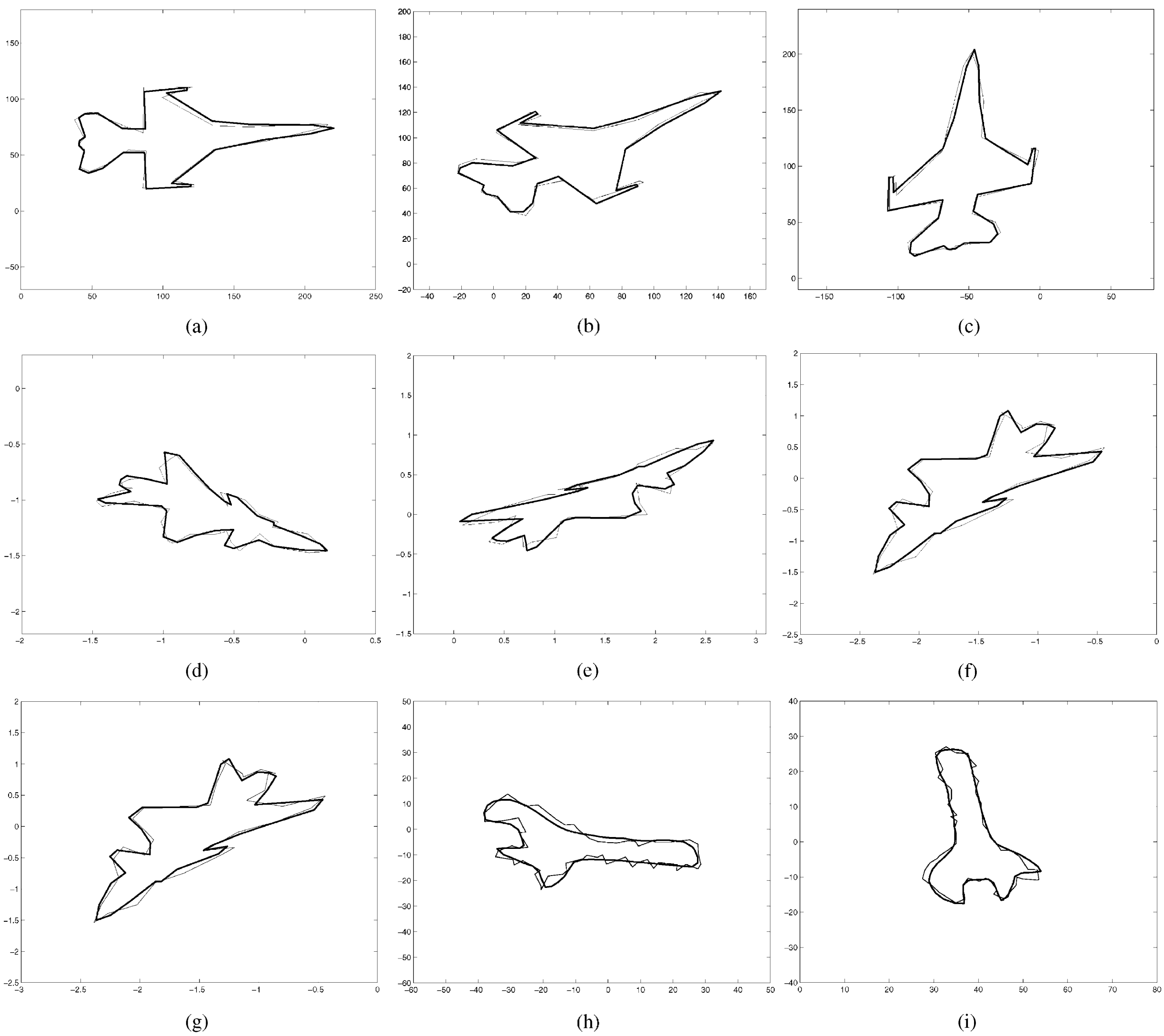


Fig. 4. Alignment results of object shapes, the thin curves are the input object shapes, the thick curves indicate the alignment results.

the increase point measure noise, but FAA outperforms the eigenvector method with higher hit rates.

From the above comparison results, we can see that FAA is superior to the traditional eigenvector method in point matching, and it is also more robust to the increasing number of points, strength of noise, number of missing and new-inserted object points.

4.2. Alignment of shapes

In this section, FAA is applied to align handwriting digits and object shapes, where digits are represented by point sets, and object shapes are denoted by the landmark points along the object boundaries.

Fig. 3 shows the alignment results of handwriting digits. The digital characters in Fig. 3(a) and (d) are considered as the references, X, and those points in Fig. 3(b), (c), (e) and (f), which are labeled with ' + ', are objects, Y. The transformational parameters are estimated by aligning the reference digits X with object digits Y using FAA, and the results are plotted in Fig. 3(b), (c), (e) and (f) by the points '○'. These points are the transformed version of the relevant reference points. From the figures, we can see that though the point numbers of the references and the objects are different, and there are local deformation to the digits, FAA still can estimate the transformational parameters and match the digits quite well.

Figs. 4 and 5 show more alignment results of plane profiles and spanner shapes using FAA. In this case, landmark points are used to character a given object shape, which can then be linked by line sections to form the boundary. In the figures, the thin curves represent the input object shapes, while the thick curves are the alignment results, that is, the transformed versions of the reference shapes using the estimated transformational parameters. Fig. 4 shows the results in different rotation, scaling, translation and shearing parameters of shapes, while in Fig. 5, more pronounced FAA's behavior is observed with the ability to align uncompleted object shapes under various positions.

In Fig. 6 different references are used to align one input shape, so that the type of the shape can be determined by the best matches with the minimum error. This realizes the process of object recognition and at the same time, localization. Fig. 6 shows the alignment results between the input shape of pliers and three references (pliers, spanner and key) using FAA. Fig. 6(a) is the input, a pair of pliers, and the alignment results using the references are shown in Fig. 6(b)–(d). The final values of the fuzzy objective function equal to 52.7, 215.4 and 280.7, respectively. Therefore, the type and location of the input shape

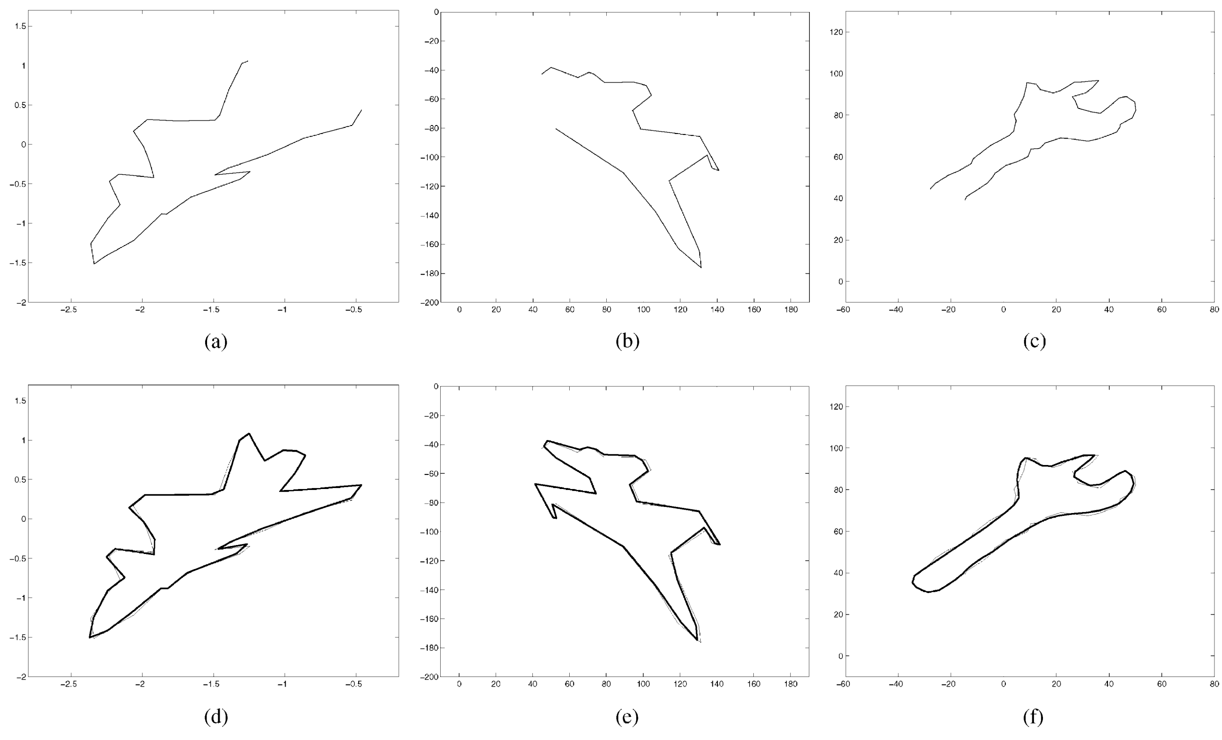


Fig. 5. Alignment results when the shapes are uncompleted, the thin curves are the input shapes, the thick curves are the alignment results.

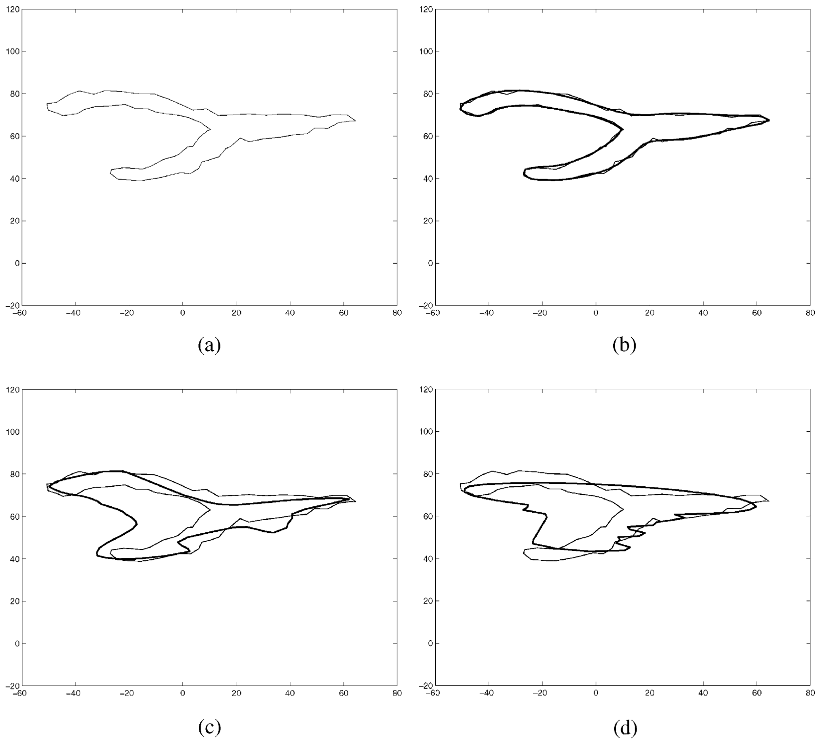


Fig. 6. Results of FAA using different references. (a) The input shape of pliers, (b) alignment result using the reference shape of pliers, the objective energy converges to 52.7, (c, d) alignment results using other prototypes (spanner and key), and the final objective energies are 215.4 and 280.7, respectively.

are obtained by Fig. 6(b), because its objective function value is much less than others.

5. Conclusion

An efficient fuzzy alignment algorithm (FAA) for aligning shapes is presented in this paper. FAA extends Marques' method to affine transformations, and it can provide an effective solution for shape alignment problem. In addition, an initialization method is designed based on affine-invariant features to improve the performance of FAA. By comparing with the eigenvector method, the effectiveness of FAA is investigated through a sensitivity study. Moreover, experiments on aligning object shapes demonstrate the good performance of FAA and indicate its application potential in the field of object detection, localization, tracking and recognition.

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About the Author—ZHONG XUE received the B.E. and M.E. degrees in Electrical Engineering from Xidian University, China in 1993 and 1996 respectively. From 1996 to 1998, he was a Research Engineer in the School of Electrical Engineering, Xidian University, and in 1998 he was a lecturer of the university. He is currently working on his Ph.D. degree in Electrical Engineering at Nanyang Technological University, Singapore. His research interests include signal and image processing, computer vision and pattern recognition.

About the Author—DINGGANG SHEN received his BS, MS and Ph.D. degrees in Electronic Engineering from Shanghai JiaoTong University (SJTU), China, in 1990, 1992 and 1995, respectively. He worked as a Research Assistant in the Hong Kong University of Science & Technology from 1994 to 1995. From 1995 to 1996, he was a Lecturer of Communication Engineering at SJTU. He was a Research Fellow in the Department of Computer Science at City University of Hong Kong from Feb. 1996 to Aug. 1997. From June 1997 to Feb 1999, he worked as a Research Fellow in School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He is presently a Postdoctoral Research Fellow in School of Medicine at Johns Hopkins University, doing medical imaging. His research interests include computer vision, pattern recognition, image processing, neural network and image indexing & retrieval. He has published over 60 articles in journals and proceedings, and is on the Editorial Board of Rapid and Brief Communication section of Pattern Recognition.

About the Author—EAM KHWANG TEOH received his B.E. and M.E. degrees in Electrical Engineering from the University of Auckland, New Zealand in 1980 and 1982 respectively and the Ph.D. degree in Electrical & Computer Engineering from the University of Newcastle, New South Wales in 1986. Presently, he is a Vice-Dean and an Associate Professor in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He is a co-inventor of an Australian Patent on An Adaptive Thickness Controller for a Rolling Mill. His current research interests are in the field of Computer Vision and Pattern Recognition, Intelligent Systems, Robotics and Industrial Automation. He has published more than 160 journal and conference papers in these areas.