Method for Calculation of Corneal Profile and Power Distribution

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• **Techniques are described for the calculation of details of corneal profile and optical power distribution along the anterior corneal surface using raw data available from an existing commercial instrument.**

(*Arch Ophthalmol* 1981;99:1261-1265)

With the introduction of various new techniques used to modify corneal shape, it is desirable to have methods for measuring both short- and long-term effects on corneal curvature or "profile," particularly when highly distorted corneas are modified. There are, of course, a number of commercial instruments that provide information about corneal curvature. One of these instruments, the corneascope, also called a "photo keratoscope" (International Diagnostic Instruments), generates data that may be used to reconstruct corneal profile. The essential part of this particular instrument is a hemispherical faceplate, which has nine concentric ring-shaped light sources. The patient's eye is located so that the cornea and the hemispherical faceplate have approximately the same center of curvature. Light from each ring is reflected from the anterior corneal surface to a lens system located on the axis of the hemisphere, and then to a camera. In a typical corneascope photograph, a series of nested rings is seen on the cornea. If the cornea is spherical (or symmetric), the rings will appear round and concentric as illustrated in Fig 1. In contrast, rings on an astigmatic cornea would appear elliptical. Rings are packed tightly toward the center when keratoconus is present, and are spread out if the cornea is flattened.

This instrument normally is used with an optical-mechanical compara-

Accepted for publication Sept 22, 1980.

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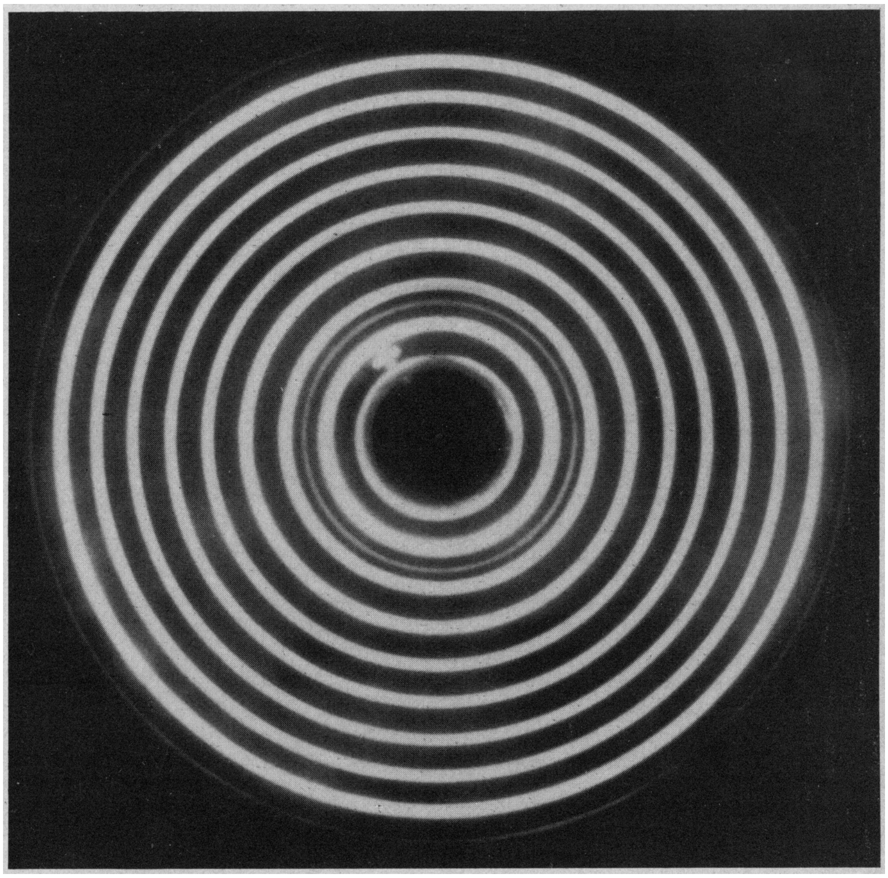


Fig 1.—Appearance of corneascope rings as photographed on surface of 7.93-mm-radius stainless steel sphere.

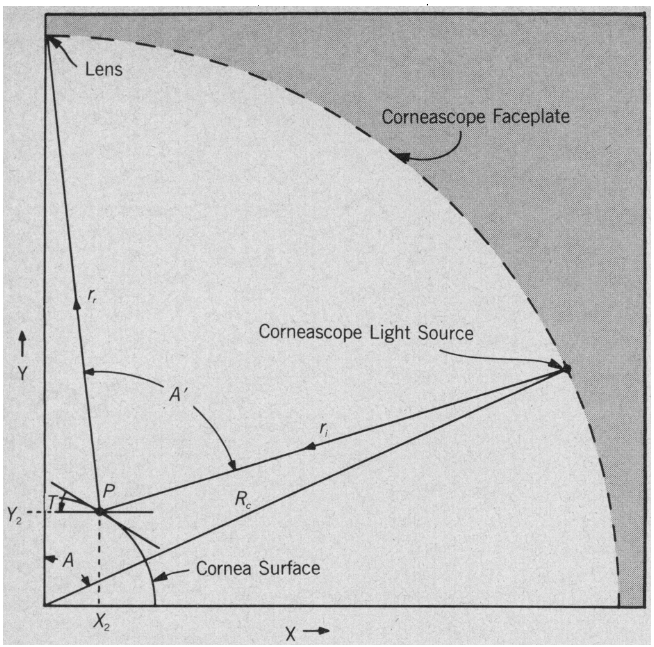


Fig 2.—Geometry used in theoretical model of corneascope. Only one of nine light sources is shown; relative diameter of "cornea" is exaggerated to facilitate illustration of details.

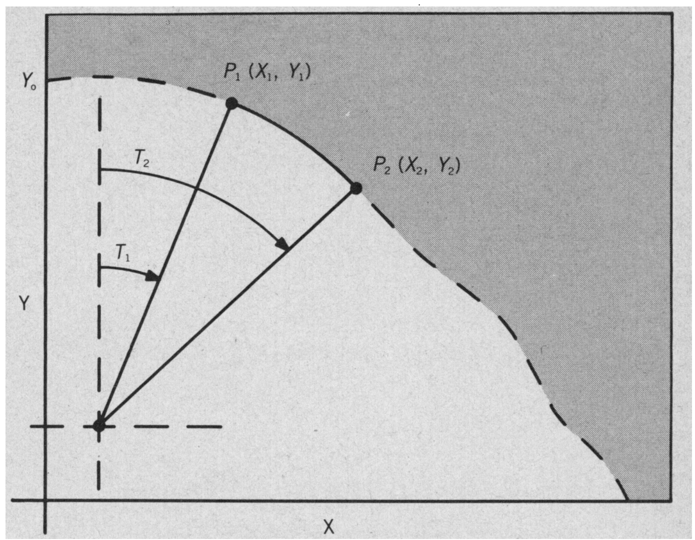


Fig 3.—Multiple-arc technique for reconstruction of corneal profile.

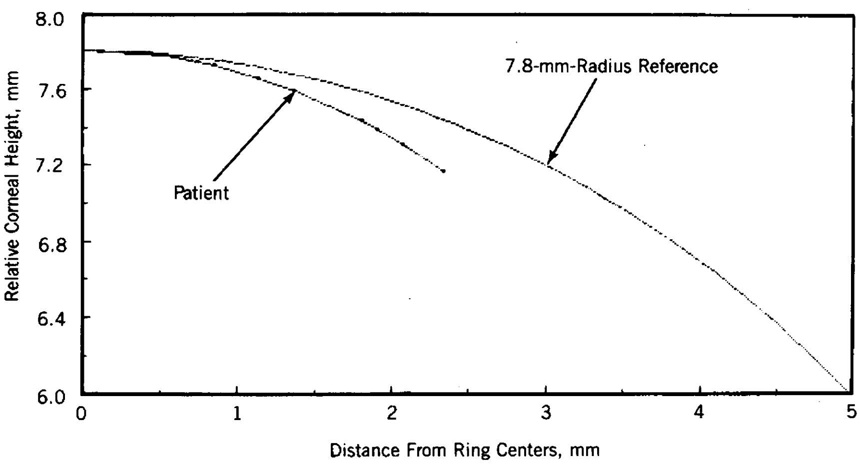
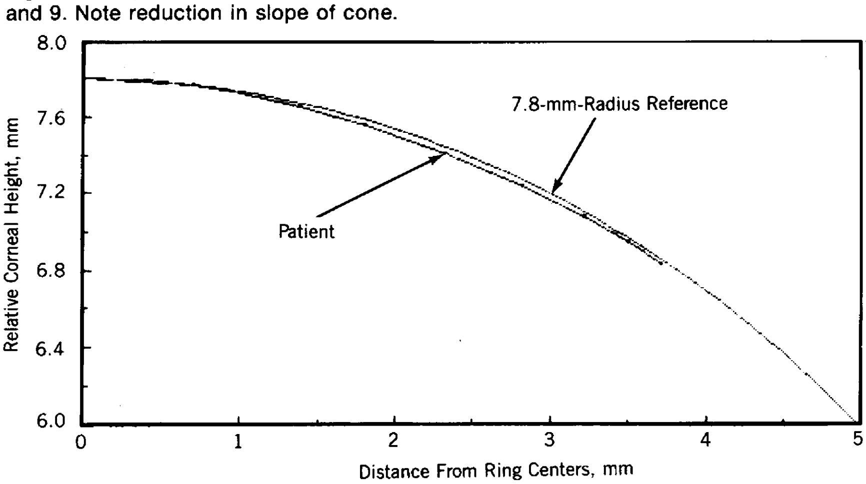


Fig 4.—Cornea profile of keratoconus patient (270° radial, OD) prior to therapy compared to profile for 7.8-mm sphere.

Fig 5.—Postoperative profile of keratoconus patient (270° radial, OD) depicted in Fig 4, 7, and 9. Note reduction in slope of cone.



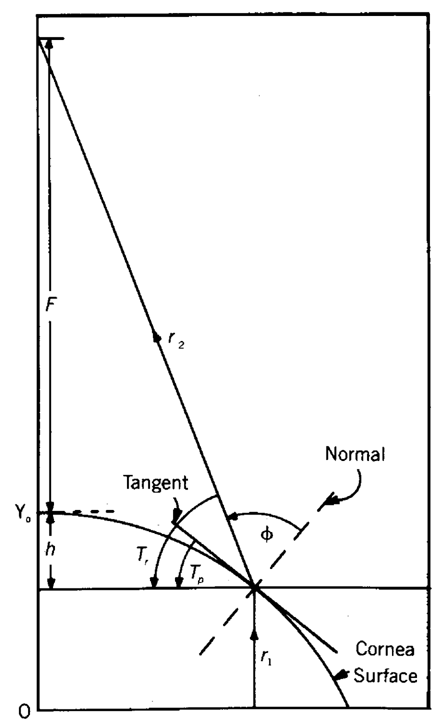


Fig 6.—Ray technique for calculating optical power at each location on cornea defined by (ring) reflection.

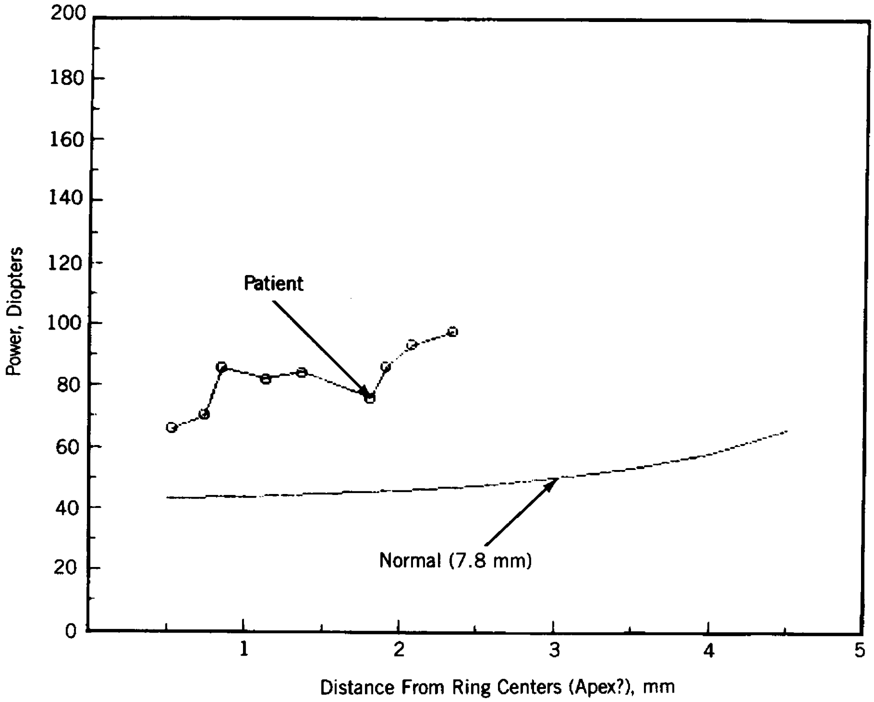
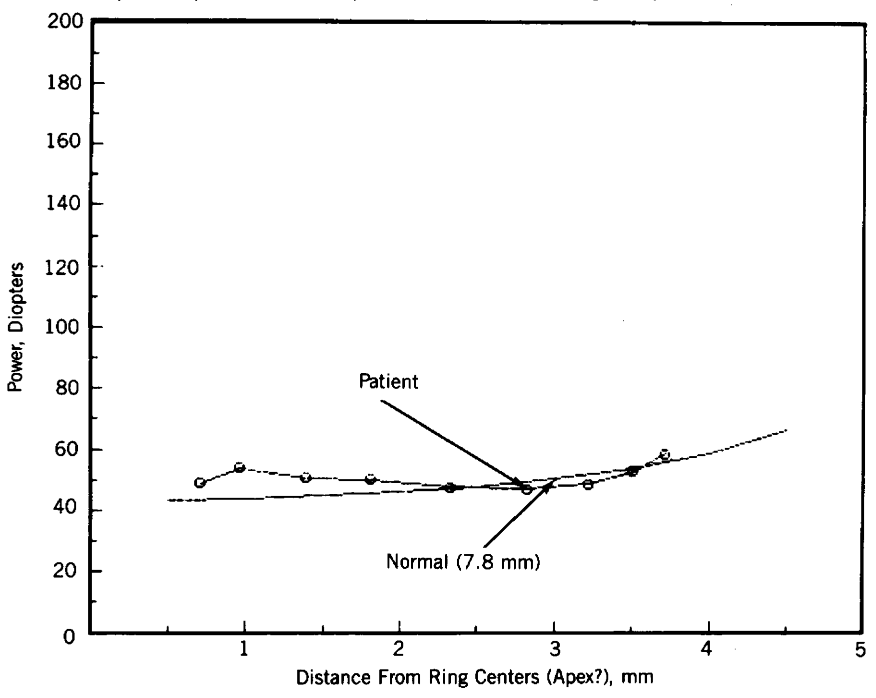


Fig 7.—Plot of patient optical power distribution (270° radial, OD) at ring reflection locations in comparison to power for 7.8-mm-radius spherical cornea. This keratoconus patient presents expected high power, as well as large degree of irregularity.

Fig 8.—Postoperative power distribution plot for keratoconus patient (270° radial, OD). Note reduction of irregularity.



tor to determine corneal radius of curvature, in the range from 9.6 to 6.34 mm, which corresponds to a range of approximately 35 to 53 diopters. We have used the corneascope to measure highly distorted corneas before and after therapy[1,2] and have found that keratoconus corneas often exhibit powers in the 80- to 120-D range, well above the upper limit of the comparator. This is not a problem in typical contact lens fitting applications (ie, the range where the manufacturer intends this instrument to be used), since contact lenses usually cannot be fitted in the advanced stages of keratoconus. More detailed information about rather distorted corneas often is useful, however, and this instrument does provide raw data that may be used to determine corneal shapes that are out of the range of its comparator.

We have developed a computational technique that expands the useful range of this corneascope by reconstruction of corneal profile and determination of optical powers along the corneal surface. Certain quantitative aspects of the code are based on corneascope dimensions and measurements from corneascope photographs of spheres of known radius. A computer program has been written in BASIC, so that the calculations may be performed on a variety of relatively inexpensive desk-top computers with minor modifications to the original code.

CALCULATION METHODS

Our theoretical model of the corneascope is shown in Fig 2. In this two-dimensional section, the left-hand vertical (Y-ordinate) axis is the axis of vision. The X-Y origin, in the lower left-hand corner, is the center of curvature for the corneascope faceplate. In this diagram a spherical cornea is shown with its center of curvature also at the origin. The size of this cornea, in relation to the corneascope radius of curvature (R_c), is exaggerated so that details of the geometry may be illustrated.

Light from the corneascope ring location, at an angle A from the Y-axis, will illuminate a large portion of the corneal surface. In our theoretical model, with a point source and pinhole lens, only one incident ray (r_i) will reflect (r_r) off the cornea and pass through the lens. (We are assuming a cornea that has a reasonably regular surface; it would be possible to have reflections through the pinhole lens from several locations if the cornea were extremely irregular.)

Angle T defines the plane surface, which is tangent to the curved surface at the point of reflection, P. If the cornea is perfectly spherical and concentric with the corneascope faceplate, T is simply equal to $A/2$. This value for T is a reasonably good

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NAME OF CODE IS CORPLT; THIS CODE CALCULATES ANGLES AFTER
ACCOUNTING FOR X AND Y POSITIONS ON THE CORNEAL SURFACE
PATIENT IS JOHN DOE
MEASUREMENT RADIAL IS 270 DEGREE
EYE MEASURED IS RIGHT
TREATMENT DATE WAS JULY 11 1979
MEASURMENT DATE IS JULY 11 1979
MAGNIFICATION FACTOR IS  1


PATIENT RING RADIUS VALUES (IN mm) APPEAR UNDER RING NUMBERS BELOW .


    1        2        3        4        5        6        7        8        9

  .536     .748     .854    1.143    1.372    1.817    1.911    2.075    2.341


BELOW, UNDER THE RING NUMBERS ARE PATIENT POWER LEVELS, FOLLOWED
BY POWER LEVELS FOR A 7.8mm RADIUS CORNEA WITH A  1.338  INDEX OF
REFRACTION AT THE SAME CORNEAL RADIUS VALUES LISTED ABOVE


RING NUMBERS (1 THRU 9):
    1        2        3        4        5        6        7        8        9

RADIAL DISTANCE FROM CENTER TO RING LOCATIONS, (mm)
  .536     .748     .854    1.143    1.372    1.817    1.911    2.075    2.341

PATIENT POWER DATA BELOW, RIGHT EYE, (DIOPTERS)
 65.925   70.258   85.842   82.161   84.473   76.061   86.290   93.595   97.779

NORMAL (7.8mm) POWER AT SAME RADIAL DISTANCES
 43.518   43.694   43.805   44.190   44.582   45.594   45.853   46.349   47.280

RADIUS OF CURVATURE BETWEEN Nth AND (N-1)th RINGS (COMPARE TO 7.8mm)
 ,5.177    4.352    1.882    5.550    4.457    9.584    1.844    2.409    5.653
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Fig 9.—Computer output above lists values of ring radius that were input by operator for this keratoconus patient. Output is optical power of anterior corneal surface at each ring location, compared to "normal" values for 7.8-mm sphere. Format is shown only for example; it is expected that each user will modify output content and format to suit special needs.

Fig 10.—Posttreatment printout for keratoconus patient. Note reduction in optical power at various ring locations in comparison with Fig 9.

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NAME OF CODE IS CORPLT; THIS CODE CALCULATES ANGLES AFTER
ACCOUNTING FOR X AND Y POSITIONS ON THE CORNEAL SURFACE
PATIENT IS JOHN DOE
MEASUREMENT RADIAL IS 270 DEGREE
EYE MEASURED IS RIGHT
TREATMENT DATE WAS JULY 11 1979
MEASURMENT DATE IS JULY 13 1979
MAGNIFICATION FACTOR IS  1


PATIENT RING RADIUS VALUES (IN mm) APPEAR UNDER RING NUMBERS BELOW


    1        2        3        4        5        6        7        8        9

  .707     .960    1.399    1.809    2.328    2.821    3.222    3.509    3.721


BELOW, UNDER THE RING NUMBERS ARE PATIENT POWER LEVELS, FOLLOWED
BY POWER LEVELS FOR A 7.8mm RADIUS CORNEA WITH A  1.338  INDEX OF
REFRACTION AT THE SAME CORNEAL RADIUS VALUES LISTED ABOVE


RING NUMBERS (1 THRU 9):
    1        2        3        4        5        6        7        8        9

RADIAL DISTANCE FROM CENTER TO RING LOCATIONS, (mm)
  .707     .960    1.399    1.809    2.328    2.821    3.222    3.509    3.721

PATIENT POWER DATA BELOW, RIGHT EYE, (DIOPTERS)
 48.830   53.672   50.524   50.040   47.608   46.892   48.700   52.628   58.561

NORMAL (7.8mm) POWER AT SAME RADIAL DISTANCES
 43.655   43.932   44.634   45.572   47.231   49.428   51.818   53.974   55.877

RADIUS OF CURVATURE BETWEEN Nth AND (N-1)th RINGS (COMPARE TO 7.8mm)
  6.986    5.248    8.447    8.085   10.817   10.639    8.366    6.026    4.341
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approximation for nonspherical surfaces if the corneal radius of curvature is small compared with the radius of curvature (R_c) of the corneascope. A more accurate determination for the angle T will be developed later.

As one views the typical corneascope photograph with nine nested rings on the patient's cornea, it is natural to use either the ring diameter or the radius as raw data for calculations. We have used the radius of individual rings, along specified corneal radii, as input data for our program. Referring again to Fig 2, X_2 is the radius of the particular ring section that is the image of the light source shown. If one is to reconstruct corneal profile, obviously it is necessary to find the value of Y_2, the corresponding vertical coordinate on the reflecting surface. A problem becomes apparent at this point: for nonspherical corneas there is insufficient information available to define a unique value of Y_2. Y_2 may vary (within the depth of field of the corneascope camera optics) without affecting the observed value of X_2 if the angle T increases as Y_2 increases. Since it is impossible to determine a unique value for Y_2 for nonspherical surfaces with the information at hand, an approximation technique is required.

The calculative technique we have used (illustrated in Fig 3) is based on a model where the corneal section comprises a sequence of short arcs that are tangentially connected at the surface reflection locations of the corneascope light sources. If X_1, Y_1, and T_1 are known for a point (P_1), and if X_2 and T_2 are known for an adjacent point (P_2), then the assumption that these two adjacent points are connected by a circular arc that has an angle T_1 at P_1 enables us to calculate Y_2 uniquely. By a sequence of such operations, a useful approximation to the corneal section profile can be constructed.

In the case of the first (innermost) ring, we have defined the reference point P_1 to be the corneal apex where $X_1 = 0$, $T_1 = 0$, and Y_1 is assigned a value of 7.8 mm at the apex. This is consistent with the alignment of the cornea around the center of curvature of the corneascope. When X_1, Y_1, X_2, and the corresponding angles of inclination (T_1 and T_2) are known, one can calculate the value of Y_2 as shown:

$$Y_2 = Y_1 - \frac{(X_1 - X_2) \cdot (\cos[T_1] - \cos[T_2])}{\sin(T_1) - \sin(T_2)}$$

In this form (X_1,Y_1) is the known point at one end of the arc corresponding to the (N-1)th ring, while (X_2,Y_2) corresponds to the location (at the other end of the arc) of the reflection point on the corneal surface of the Nth ring. In the same manner, T_1 and T_2 are the angles of inclination (T), for the (N-1)th and Nth rings, respectively.

At each ring location, Y_2 is first calculated as shown above, where T_2 is set equal to $A/2$. Following this initial calculation of the value of Y_2, the value of T_2 is refined by first using the estimated value of Y_2 to calculate the angle A', which is enclosed by the incident and reflected rays (r_i and r_r) shown in Fig 2.

A' has the value* shown below, where ($x = X_2$ and $y = Y_2$):

$$A' = \cos^{-1}\{[4R_c^2 \cdot \sin^2(\tfrac{A}{2}) - ([R_c - y]^2 + x^2) - ([R_c \cdot \sin\ A - x]^2 + [R_c \cdot \cos\ A - y]^2)] / [-2 \cdot \sqrt{(R_c - y)^2 + x^2} \cdot \sqrt{(R_c \cdot \sin A - x)^2 + (R_c \cdot \cos A - y)^2}]\}$$

The refined value of T is referred to as T_p, and

$$T_p = 90° - \frac{A'}{2} - \tan^{-1}\{(R_c \cdot \cos\ A - y)/(R_c \cdot \sin A - x)\}$$

This new value is now assigned to T_2, and Y_2 is calculated once more with somewhat greater precision. (One may iterate through this procedure a number of times until Y_2 is stabilized to some preset requirement; we have found that one operation is sufficient for our needs.) In addition, the refined values for T_2 (ie, T_p) and Y_2 will be used for T_1 and Y_1, respectively, during the calculation of Y_2 for the next ring.

In this manner the surface reflection locations (X_2, Y_2) for each ring are calculated until a profile is defined from the corneal apex through the Nth (ninth) ring. Examples of this profile are shown in Fig 4 and 5, where a keratoconus patient's corneal curvature is compared with that for a 7.8-mm sphere.

In addition, we have found it useful to extend the calculation to determine details of optical power along the cornea. This calculation is accomplished by ray tracing, as illustrated in Fig 6. A ray, r_1, is assumed to emerge from the anterior corneal surface; the internal path of this ray is parallel to the Y-axis. The exit angle (T_r) of ray r_2 is related to the index of refraction (I_r) of the cornea (taken as 1.338 in our calculations) and the inclination angle (T_p) at the reflection point being considered, ie:

$$T_r = T_p + 90° - \sin^{-1}(I_r \cdot \sin[T_p])$$

The focal length (F) referred to a plane that is tangent to the corneal apex is

$$F = X_2 \cdot \tan(T_r) - (Y_o - Y_2)$$

Since focal length is expressed in millimeters in our code, power (P) at the same location is

$$P = 1{,}000/F \text{ D}$$

Plots of optical power vs radial distance from the corneal apex are illustrated in Fig 7 and 8 for the keratoconus cornea shown in profile in Fig 4 and 5. Figures 9 and 10 are examples of computer printouts of this same optical power information, which may be used in lieu of graphics.

*A' is determined by the "Law of Cosines" where (refer to Fig 2)
$r_i^2 = (R_c \cdot \sin A - x)^2 + (R_c \cdot \cos A\text{-}y)^2$
and $r_r^2 = (R_c - Y)^2 + x^2$ and the chord connecting r_i and r_r at the corneascope faceplate has the value $2 \cdot R_c \cdot \sin(\frac{A}{2})$. Thus,
$\cos A' = (4R_c^2 \sin^2(\frac{A}{2}) - r_r^2 - r_i^2)/(-2\ r_r r_i)$

COMMENT

A computer program that uses corneascope data has been written to calculate corneal profile and optical power at various points along a cornea. Since the code is in BASIC, it is adaptable to a wide variety of relatively inexpensive desk-top computers. The graphics illustrated in Fig 4, 5, 7, and 8 are available on the Hewlett-Packard 9845T desk-top computer, which was used in the development of this program. Different graphics subroutines would be used with other computers.

The accuracy of the calculations is primarily a function of the precision with which the corneascope output (ring radii) is measured, although refinement of certain parameters in the computer code that define the corneascope may also increase accuracy. The majority of our measurements are made directly from the corneascope photograph, where the cornea is magnified by a factor of 4.8. With this magnification, precision of micrometer measurement of the rings results in overall accuracy of 2% to 5%, depending on ring diameter. For example, ring radius was measured by micrometer on a corneascope photograph of a 9.52-mm-radius sphere. The calculated mean radius of curvature of the nine arcs was 9.663 mm, with a SD of 0.2385 mm. Errors in the power calculation ranged from −4.7% at ring 1 to −2.7% at ring 5, and −3.0% at ring 9. When the image is magnified further, by projection as a slide or transparency, increased measurement accuracy is possible. We have also used a Hewlett-Packard 9847A digitizer for the measurement and direct input of ring radii to the computer code. It appears that this method, with sufficient magnification, is probably capable of at least 0.5% overall accuracy. Future refinements in the techniques for measuring ring radii will certainly improve the accuracy of the final calculations of corneal profile and power distribution.

According to a personal communication from Ralph Dahlstrom, MD, Los Alamos Medical Center, July 1980, he has already adapted portions of this code (excluding graphics) to his office computer to extend the useful range of his corneascope. Output from the program has also been used at the Dean McGee Eye Institute to evaluate early clinical trials of a thermokeratoplasty technique[1,2] and is being adapted at that location for routine evaluation of corneascope data.

This research was supported by the Division of Biomedical and Environmental Research, Contract W7405-ENG-36, US Department of Energy.

A listing of either graphics or nongraphics versions of this program is available on written request to either Mr Doss or Dr Hutson, MS-844, Los Alamos Scientific Laboratory, Los Alamos, NM 87545.

The corneascope (or photo keratoscope) is manufactured by International Diagnostic Instruments, 9712 E 55th Pl, Tulsa, OK 74145.

References

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2. Doss JD, Albillar JI: A technique for the selective heating of corneal stroma. *Contact & Intraocular Lens Med J* 1980;6(Jan/March):13-17.